Griffith UNIVERSITY
Queensland, Australia

Griffith Business School


*Economics and business statistics*

# A Multi-Sector Model of the United Kingdom: Theory, Data and Parameters

**George Verikios, Kevin Hanslow, Daniel Bahyl and Reza Gharibnavaz**

2020-02

**Series editors** Dr Nicholas Rohde and Dr Athula Naranpanawa **ISSN** 1837-7750

Department of Accounting, Finance and Economics

# A Multi-Sector Model of the United Kingdom: Theory, Data and Parameters

George Verikios*

KPMG Economics and Department of Accounting, Finance and Economics, Griffith University

Kevin Hanslow, Daniel Bahyl and Reza Gharibnavaz

KPMG Economics

Keywords: computable general equilibrium, multi-sectoral, economic growth, productivity analysis, regional analysis.

**Acknowledgements**

We thank Ken Clements for helpful comments on the paper. This work was partly funded by KPMG UK.

* Email: g.verikios@griffith.edu.au.

## 1. Introduction

This document describes the theory, data and parameters of a multi-sectoral model of the four countries of the United Kingdom – England, Scotland, Wales and Northern Ireland. The four countries are represented as separate economies linked by inter-regional trade flows. The model belongs to the computable general equilibrium (CGE) class of models exemplified by the MONASH model of Australia (Dixon and Rimmer, 2002) and the multi-country GTAP model (Hertel and Tsigas, 1997). The most recent example of a CGE model of the UK is the single region model developed by HM Revenue & Customs (HMRC, 2013).

The model described here is dynamic . That is, the theory of the model refers to how model variables relate to each other within a period or across time periods. A dynamic simulation of the effects of a policy change involves running the model twice to generate the baseline and policy simulations. The baseline may be a plausible forecast of how the economy will evolve over time in the absence of the policy shock of interest. As such, the baseline may incorporate external forecasts for key macroeconomic variables in the short term with convergence to a balanced growth path in the long term. Alternatively, the baseline may represent the movement from a non-balanced to a balanced growth path via the application of balanced growth shocks to the model. With the exception of the policy variables of interest (e.g., tax rates, technology, etc.), all exogenous variables in the policy simulation are assigned the values they had in the baseline simulation. The differences in the values of variables in the baseline and policy simulations quantify the effects of moving the variables of interest away from their baseline values, i.e., the deviations of variables from their baseline values caused by the policy shock modelled.

The model distinguishes 127 sectors and commodities (see Table 2) based on the 2013 input-output (IO) tables published by the Office for National Statistics (2016). Primary factors are distinguished by 127 types of capital (one type per industry), 9 occupations, owner-operator labour (i.e., self-employed workers) and land. The national IO table is disaggregated into regional IO tables using a combination of industry shares in employment or labour hours and commodity-specific consumption shares to split industries, investment and government and private consumption across regions.

A representative firm in each sector produces a single commodity. Each commodity is distinguished between a variety destined for export markets and a variety destined for domestic sales.

Some commodities produced for use in the domestic market are further divided into a margin and non-margin component. The margin component of a commodity is used to facilitate the movement and sale of both imported and domestic commodities within the UK, and of the exported commodities to the point of exportation. Margin commodities include such activities as the various modes of transportation, and wholesale and retail trade. The non-margin component is used as a direct input into industry activity, investment and government or private consumption across all regions, or as a change in inventories within the region of production.

Production technology is represented by nested CRESH functions (Hanoch, 1971) allowing a high degree of flexibility in the parameterisation of substitution and technology parameters. Energy goods are treated separately to other intermediate goods and services in production and are complementary to primary factors.

The supply of labour within each region is determined by working-age population and a labour-leisure trade-off that allows workers in each occupation to respond to changes in after-tax wage rates thus determining the hours of work they offer to the labour market. Working-age population typically moves with population.

Household consumption decisions are determined by a linear expenditure system (Stone, 1954) that distinguishes between subsistence (necessity) and discretionary (luxury) consumption.

The model represents detailed central and local government fiscal accounts including the accumulation of public assets and liabilities. On the revenue side, detailed modelling of all direct and indirect taxes and income from government enterprises is included. On the expenditure side, government consumption, investment and payments of various types of transfers (such as pensions and unemployment benefits) are modelled.

Investment behaviour is industry specific and is positively related to the rate of return on capital. This rate takes into account company taxation and a variety of capital allowances.

Foreign asset and liability accumulation is explicitly modelled for each region, as are the cross-border income flows they generate and that contribute to the evolution of the current account. Along with other foreign income flows like labour payments and unrequited transfers, it takes account of primary and secondary income flows in the UK's current account. These are particularly important for the UK as they typically comprise a significant share of the balance on the current account.

## 2. Theory

The model is represented by equations specifying behavioural and definitional relationships. Formally, the model theory is represented by nonlinear equations specifying behavioural and definitional relationships as

$$F_i\left(\boldsymbol{N},\boldsymbol{X}\right)=\mathbf{0}, \tag{1}$$

where $F_i$ are $i$ (=1,...,$m$) continuous and differentiable functions, $\boldsymbol{N}$ is a $m\times 1$ vector of endogenous variables and $\boldsymbol{X}$ is a $n\times 1$ vector of exogenous variables. Typically, $\boldsymbol{X}$ describes changes in economic structure and policy (e.g., tariff rates, technology, etc.) and can be used to perturb the model to simulate changes in $\boldsymbol{N}$.

Solving the functions underlying (1) in nonlinear form can be computationally burdensome. To avoid this, we can approximate the changes in $\boldsymbol{N}$ by calculating the differentials of $\boldsymbol{N}$ as a function of the differentials in $\boldsymbol{X}$. To do this, we totally differentiate the nonlinear functions of equation system (1) and rearrange to obtain

$$\Delta\boldsymbol{N} = A^{-1}B\Delta\boldsymbol{X}\ , \tag{2}$$

where $A$ is an $m\times$ nonsingular matrix of coefficients (the partial derivatives of the functions $F_i$ with respect to $\boldsymbol{N}$) and $B$ is an $m\times n$ matrix of coefficients (the partial derivatives of the functions $F_i$ with respect to $\boldsymbol{X}$). Equations (2) allow us to write the changes (or percentage changes) in the endogenous variables as linear functions of the changes in the exogenous variables, significantly reducing the computational task.

As (2) is derived via total differentiation, it is an approximation based on marginal changes in $\boldsymbol{X}$. For marginal changes in $\boldsymbol{X}$ the approximation is accurate but for discrete changes in $\boldsymbol{X}$ the approximation will be inaccurate. The problem of accurately calculating $\boldsymbol{N}$ for large changes in $\boldsymbol{X}$ is equivalent to allowing the coefficients of the $A$ and $B$ matrices to be nonconstant. This is achieved by breaking the total change in $\boldsymbol{X}$ into a series of marginal changes and applying multistep solution algorithms.[1]

## 3. Production of commodities

[1] The model is implemented and solved using the multistep algorithms available in the GEMPACK economic modelling software (Harrison and Pearson, 1996).

Within each region (or country), each industry produces a single commodity that is allocated between an exported and local (that is, to be eventually used within the UK) variety via a constant elasticity of transformation (CET) frontier. Letting *COM* be the set of commodities and *REG* be the set of regions, transformation between exported and locally-used commodities is expressed as:

$$Q_{jr} = B\left[\left(\chi_{jr} Q_j^{DOM}\right)^{-\rho} + \left(1-\chi_{jr}\right)\left(Q_{jr}^{EXP}\right)^{-\rho}\right]^{-1/\rho},\ B>0, 0<\chi_{jr}<1, \rho \le -1; \quad (3)$$

where $Q_{jr}$ is the activity level or output of industry *j* in region *r*, $Q_{jr}^{DOM}$ is the quantity of the local commodity, $Q_{jr}^{EXP}$ is the quantity of the exported commodity, and $\chi_{jr}$ and $\rho$ are parameters. The CET elasticity of transformation is $\sigma_j^{DOMEXP} = 1/(1+\rho)$ and is typically set equal to a value of 20. An implication of (3) is that changes in domestic prices are not fully passed on to export prices via accommodating movements in $Q_{jr}^{EXP}$.

Some of the local commodity may be added to inventories within the region of production, or may be supplemented by a drawdown of inventories in the region of production. Any such adjustment in inventories is an exogenously imposed change under the normal model closures. The local commodity may have a margin and non-margin component. A part of each of these components is exported to other regions and the remainder of each of these components is combined into a constant elasticity of substitution (CES) composite with inter-regional imports to form the margin and non-margin components of the domestic commodity used within the region.

The price $P_{jr}^{LOC}$ is the price of the local commodity produced within the region. Nevertheless, as a consequence of inter-regional trade, $P_{jr}^{LOC}$ is not the basic price[2] of the domestic commodity used within the region. This latter price will be a CES composite of the value of $P_{jr}^{LOC}$ from all regions. This distinction between the price of the local commodity and the basic price of the domestic commodity used holds for both margin and non-margin usage.

A more formal presentation of inter-regional trade, with a discussion of data issues, is presented in section 13.

## 4. Purchasers' prices

The basic price of a domestic non-margin commodity used within a region is not necessarily the final price paid by a user of the commodity. This final price is called the purchasers' price and is constituted from the basic price, taxes levied on the basic value of the commodity, the cost of margins used to convey the commodity to the user and the VAT levied as a rate on the total value of all other components of the purchasers' price. The purchasers' price $P_{iur}^{PUR}$ is defined as

$$P_{iur}^{PUR} = P_{iur}^{PREVAT}\left(1+T_{iur}^{VAT}\right), \qquad i \in COM, u \in USR, r \in REG \quad (4)$$

where $P_{iur}^{PUR}$ is the purchasers' price of commodity *i* for user *u* in region *r*, $P_{iur}^{PREVAT}$ is the pre-VAT price of commodity *i* for user *u* in region *r*, $T_{iur}^{VAT}$ is the VAT rate applied to commodity *i* for user *u* in region *r*. *USR* is a set made up of intermediate or investment usage by each industry, private consumption and local or central government consumption.

The pre-VAT price $P_{iur}^{PREVAT}$ is defined as

[2] The basic price is the price that is received by the supplier (or producer) of the commodity; hence it is also referred to as the supply price. This price covers the producer's costs including any taxes on production.

$$P_{iur}^{PREVAT} = P_{iur}^{BAS}\left(1+T_{iur}^{BAS}\right) + \sum_{m\in MAR} S_{iumr}^{MAR}\left(P_{ir}^{MAR}\cdot A_{imr}^{MAR}\right)$$

$$i\in COM,\ u\in USR,\ r\in REG \qquad (5)$$

where $P_{ir}^{BAS}$ is the basic price of commodity *i* in region *r*, $T_{iur}^{BAS}$ is the tax rate applied to the basic value of commodity *i* for user *u* in region *r*, $P_{mr}^{MAR}$ is the basic price of margin *m* in region *r*,[3] $A_{imr}^{MAR}$ is the per unit input requirement for margin *m* conveying commodity *i* in region *r*, and $S_{iumr}^{MAR}$ is the share of margin *m* used to convey commodity *i* to user *u* in region *r*.

The expression $\left(P_{mr}^{MAR}\cdot A_{imr}^{MAR}\right)$ is the *effective* price of margin *m* for conveying commodity *i* in region *r*. Note that $\Delta A_{imr}^{MAR} < 0$ means a fall in the per unit input requirement, that is, technical improvement or progress. This means that for a given $P_{mr}^{MAR}$, $\Delta A_{imr}^{MAR} < 0$ means $\Delta\left(P_{mr}^{MAR}\cdot A_{imr}^{MAR}\right) < 0$, that is, a fall in the *effective* price. This is true for all per unit requirement variables presented below.

The tax levied on the basic value of a commodity $T_{iur}^{BAS}$ may be constituted from many different taxes levied on the use of intermediate inputs to production, as described in section 5.

The demand for a margin is modelled as the quantity of commodity being conveyed times the per unit requirement for the margin, that is:

$$Q_{iumr}^{MAR} = A_{imr}^{MAR} Q_{iur}\,. \qquad i\in COM,\ u\in USR,\ r\in REG,\ m\in MAR \qquad (6)$$

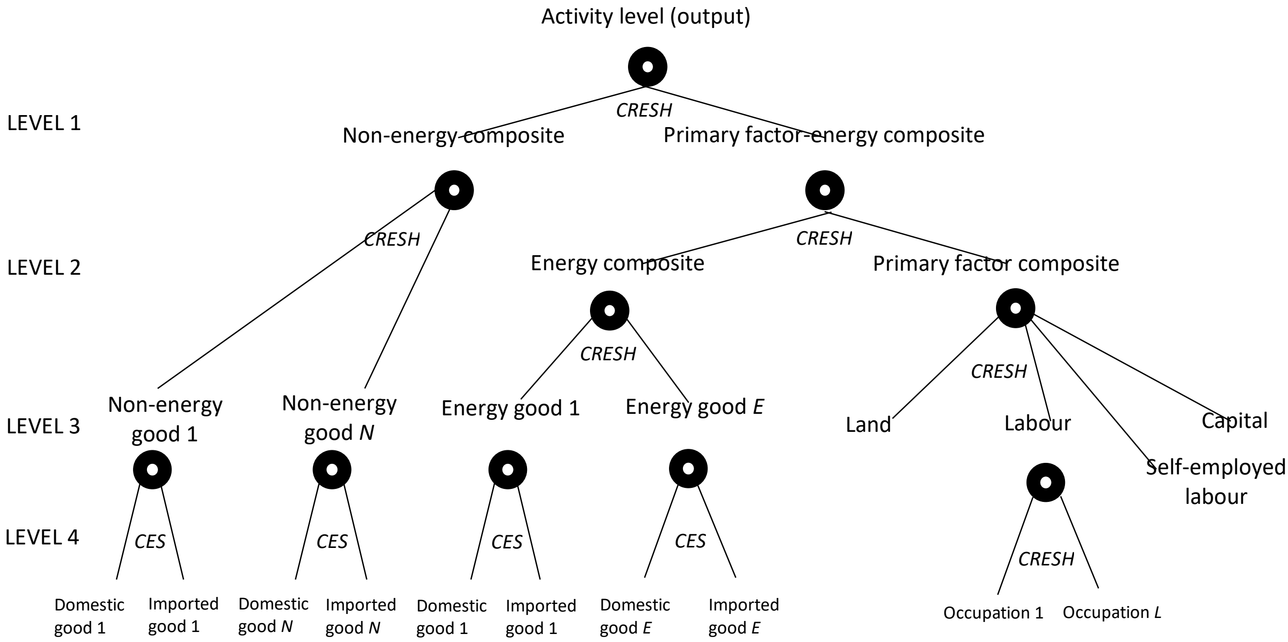
Equations of identical structure to (4) and (5) define the purchasers' price for each imported commodity in terms of taxes, margins and the basic price of the commodity. For an imported commodity the basic price is the landed duty-paid price, which is equal to the domestic currency CIF price[4] times the power of the import tariff rate, i.e., 1 + import tariff rate.

## 5. Input technology: current production

Within each region a representative firm in each sector produces a single commodity.[5] The model recognises two broad categories of inputs: intermediate inputs and primary factors. Representative firms choose inputs of primary factors and intermediate inputs to minimise costs subject to a given production technology and given factor and commodity prices. Primary factors include land, nine types of labour (occupations),[6] owner-operator labour and physical capital. Intermediate inputs consist of 127 domestically-produced goods and services and 127 foreign substitutes. Demands for primary factors and intermediate inputs are modelled using nested production functions. As apparent from Figure 1, the nested production functions , which define the production technology available to the representative firm, have four tiers.

[3] There are no taxes on margins, or margins on margins, so there is no distinction between the basic price and purchasers' price of a margin.
[4] The price inclusive of cost, insurance and freight.
[5] All the variables in this and most subsequent section have a region subscript but it has been dropped to reduce notational clutter, i.e., the equations are replicated in all regions.
[6] The occupational classification corresponds to 1-digit occupations in ONS (2010).

**Figure 1. Input technology for current production**




## 5.1. Level 1: Non-energy composite and primary factor-energy composite

At the top level firms determine optimal quantities of the non-energy composite (i.e., an aggregate of non-energy intermediate inputs) and the primary factor-energy composite (i.e., an aggregate of primary factors and energy intermediate inputs) subject to a CRESH (constant ratios of elasticities of substitution, homothetic) production technology. This formulation relaxes the assumption implied by CES functions that the elasticity of substitution across all pairs of inputs must be the same. CRESH production functions were introduced as a generalisation of CES by Hanoch (1974). The nested CRESH functional form is more desirable than alternative techniques such as fixed coefficients (Leontief) production technology since it allows us to take advantage of differences in econometrically-estimated values of the elasticities of substitution across individual factors.

The production technology distinguishes between primary and intermediate factors of production. Profit maximising producers are capable of choosing the optimal combination of primary factors independently of the prices of intermediate inputs. The nested CRESH functions allow different elasticities of substitution to exist between primary factors of production and goods. Thus, the optimal mix of the primary factor-energy composite (*PF-E*) and non-energy composite (*NE*) is represented as

$$\sum_i \left[\frac{Q_{ij}^F}{Q_j A_{ij}^F}\right]^{h_i} \frac{X_i}{h_i} = \alpha, \qquad 0 < h_i < 1, X_i > 0, \sum_i X_i = 1, \alpha > 0\ ,$$

$$i \in PF\text{-}E,\ NE,\ j \in IND. \tag{7}$$

In (7) $Q_{ij}^F$ is the demand for factor *i* by industry *j*, $A_{ij}^F$ is factor-specific unit input requirements, and $X_i$, $h_i$ and $\alpha$ are parameters. The elasticity of substitution between the primary factor-energy composite (*i*=*PF-E*) and non-energy composite (*j*=*NE*) is $\sigma_i\sigma_j \big/ \sum_{k=1}^{2} S_k\sigma_k$ where $\sigma_i = 1/1 - h_i$ is the CRESH parameter associated with input *i* and $S_k$ is the k-*th* input's cost share. In the special case when all $\sigma_i$ have the same value, the CRESH system is equivalent to CES and all substitution elasticities are equal.

The elasticity of substitution between non-energy and primary factor-energy composites is 0.1 for all industries. Adopting an elasticity of substitution of 0.1 at this level of the production nest assumes that firms' use of the non-energy composite and the primary factor-energy composite is close to a fixed share of output. This reflects the idea that the output share of these two composites is nearly invariant to changes in relative prices (i.e., they are complements) and reflects characteristics intrinsic to the production of each good. Note that these shares will vary if there is a change in production technology, e.g., innovation that allows less use of non-energy inputs per unit of output.

### 5.2. Level 2: Energy composite and primary factor composite

At the second level of the production nest, firms choose the optimal mix of the energy (*E*) and primary factor (*PF*) composites. The energy composite is an aggregation of energy intermediate inputs; the primary factor composite is an aggregation of all primary factors. These two composites are combined using CRESH production technology

$$\sum_i \left[ \frac{Q_{ij}^{PF-E}}{Q_{kj}^{F} A_{ij}^{PF-E}} \right]^{m_i} \frac{Y_i}{m_i} = \varepsilon , \qquad 0 < m_i < 1, Y_i > 0, \sum_i Y_i = 1, \varepsilon > 0 ,$$

$$i=PF,\ E,\ k=PF\text{-}E,\ j \in IND. \tag{8}$$

where $Q_{ij}^{PF-E}$ is demand for factor *i* (=*PF*, *E*) by industry *j*, $A_{ij}^{PF-E}$ is factor-specific unit input requirements, and $Y_i$ , $m_i$ and $\varepsilon$ are parameters. The elasticity of substitution between the energy composite (*i*=*E*) and primary factor composites (*j*=*NE*) is $\sigma_i \sigma_j \big/ \sum_{k=1}^{2} S_k \sigma_k$ where $\sigma_i = 1/1 - m_i$ is the CRESH parameter associated with input *i* and $S_k$ is the k-*th* input's cost share. The elasticity of substitution is set equal to 0.1. This choice of parameters effectively makes the energy and primary factor composites near complements.

### 5.3. Level 3: Non-energy inputs, energy inputs and primary factors

At the third level of the production nest, firms choose cost-minimising combinations of constituents in each of the non-energy intermediate inputs composite (*NE*), energy intermediate inputs composite (*E*) and primary factor composite (*PF*).

The optimal mix of non-energy intermediate inputs is chosen subject to CRESH production technology

$$\sum_i \left[ \frac{Q_{ij}^{NEI}}{Q_{kj}^{F} A_{ij}^{NEI}} \right]^{y_i} \frac{Z_i}{y_i} = \eta , \quad 0 < y_i < 1, Z_i > 0, \sum_i Z_i = 1, \eta > 0 ,$$

$$i \in NEI,\ k=NE,\ j \in IND, \tag{9}$$

where $Q_{ij}^{NEI}$ is demand for non-energy input *i* ($\in$*NEI*) by industry *j*, $Q_{kj}^{F}$ (*k*=*NE*) is demand for the non-energy composite, $A_{ij}^{NEI}$ represent unit input requirements for non-energy inputs and $Z_i$ , $y_i$ and $\eta$ are parameters. The elasticity of substitution across non-energy inputs is $\sigma_i \sigma_j \big/ \sum_{k \in NEI} S_k \sigma_k$ where $\sigma_i = 1/1 - y_i$ is the CRESH parameter associated with input *i* and $S_k$ is the k-*th* input's cost share. The elasticity of substitution across all pairs of non-energy intermediate inputs is 0.25 for all industries based on estimates by Bruno (1984) and Atalay

(2017). These values imply that firms have some choice with respect to energy technology and will alter the pattern of energy usage in production if relative prices change.

Analogously, the optimal mix of energy intermediate inputs $Q_{ij}^{EI}$ $i$ ($\in EI$) is determined subject to CRESH production technology (viz. equation (9)) with an elasticity of substitution across all pairs of energy intermediate inputs of 0.25 for all industries. Thus firms also have some choice with respect to non-energy technology and will alter the pattern of non-energy usage in production if relative prices change.

At this level of the production nest firms also determine the optimal mix of capital and the land and labour composites subject to CRESH technology

$$\sum_i \left[ \frac{Q_{ij}^{FAC}}{Q_{kj}^{PF-E} A_{ij}^{FAC}} \right]^{n_i} \frac{L_i}{n_i} = \pi , \quad 0 < n_i < 1, L_i > 0, \sum_i L_i = 1, \pi > 0 ,$$

$$i \in FAC,\ k{=}PF,\ j \in IND, \tag{10}$$

where $Q_{ij}^{FAC}$ is demand for primary factor $i$ ($\in FAC$) by industry $j$, $Q_{kj}^{PF-E}$ ($k$=$PF$) is demand for the primary factor composite, $A_{ij}^{FAC}$ are unit input requirements for primary factor $i$, $L_i$, $n_i$ and $\pi$ are parameters The elasticity of substitution across primary factor is set to 0.5 based on the survey by Chirinko (2008).

### 5.4. Level 4: Domestic inputs, imported inputs, labour and land types

At the lowest level of the production nest, firms decide on the optimal mix of domestic (*DOM*) and foreign (*IMP*) intermediate inputs subject to CES technology. For non-energy intermediate inputs this is represented as

$$Q_{ij}^{NEI} = \left[ \sum_s \mu_s \left( \frac{Q_{isj}^{INT}}{A_{isj}^{INT}} \right)^{-\tau_i} \right]^{-1/\tau_i} , \quad 0 < \mu_s < 1, \sum_s \mu_s = 1, \tau_i \geq -1, \tau_i \neq 0 ,$$

$$i \in NEI,\ s \in SRC,\ j \in IND. \tag{11}$$

In (11) $Q_{ij}^{NEI}$ is demand for non-energy composite $i$ by industry $j$, $Q_{isj}^{INT}$ is demand for non-energy commodity $i$ from source $s$ ($\in SRC$, $SRC$=$DOM$, $IMP$) by industry $j$, and $A_{isj}^{INT}$ are input-specific unit input requirements. $\mu_s$ and $\tau_i$ are parameters. The CES elasticity of substitution is $\sigma_i = 1/(1+\rho_i)$. There is an equivalent set of equations to (11) representing the combination of energy intermediate inputs ($i \in EI$) by source.

The values of $\sigma_i$ are drawn from econometric estimates based on UK data over the period 1998 to 2016. This work is described in Section 0. Our elasticity estimates imply low to medium responsiveness of firms to relative price changes between domestic and foreign goods. Thus, the elasticities of substitution range from 1.5 for primary goods, between 1.1 and 1.8 for processed food, 2.5 for textile, clothing and leather products, 3.8 for chemical products, and around 1 for most other manufactured goods. See section 0 for a complete listing of these values. The elasticities are zero for most services, the exceptions being water and air transport that use a value of 2.

At this level, firms also choose the optimal mix of the $o$ ($\in OCC$, $OCC$=1,..,9) labour types (i.e., occupations) subject to CRESH technology

$$\sum_o \left[ \frac{Q_{oj}^{OCC}}{Q_{kj}^{F} A_{oj}^{OCC}} \right]^{v_i} \frac{X_o}{v_o} = \lambda , \qquad 0 < v_i < 1, X_i > 0, \sum_o X_o = 1, \lambda > 0 ,$$

$$o \in OCC,\ k \in LAB,\ j \in IND, \qquad (12)$$

where $Q_{oj}^{OCC}$ is demand for occupation *o* by industry *j*, $Q_{kj}^{FAC}$ (*k*=*LAB*) is demand for the labour composite, and $A_{oj}^{OCC}$ represents unit input requirements. The elasticity of substitution across occupations is $\sigma_i \sigma_j \big/ \sum_{o=1}^{9} S_o \sigma_o$ where $\sigma_i = 1/1 - v_i$ is the CRESH parameter associated with occupation *i* and $S_o$ is the o-*th* occupation's cost share. The elasticity of substitution is set to 0.25 representing limited possibilities for substitution across occupations.

At level 4 firms also decide on their use of two land types (primary production land and non-primary production land) using CRESH technology. At this stage, it is assumed that each industry uses only one type of land and that this cannot change. Thus, the elasticity of substitution between land types is set to zero for all industries and individual land usage moves with demand for the land composite ($Q_{kj}^{FAC}$ , *k*=*LND*). Consistent with this assumption, although the model data can be aggregated (e.g., to reduce the size of the model or to reduce sectoral detail when it is not required) the primary production industries are never aggregated with the non-primary production industries.

## 6. Zero-pure-profits and market clearing

All firms are assumed to operate in competitive markets and thus take their output prices as given. Consistent with this we impose a zero-pure-profits condition that equates revenues with costs and determines each industry's activity level or output:

$$P_i^{BAS} Q_j = \sum_{k \in COM} \sum_{s \in SRC} P_{ksj}^{INT} Q_{ksj}^{INT} + \sum_{f \in FAC} P_{fj}^{FAC} Q_{fj}^{FAC} , \quad i \in COM, j \in IND. \qquad (13)$$

In (13), the left-hand side is revenue for the *j*-th industry comprising the product of the basic price of *i*-th commodity $P_i^{BAS}$ and the output of the *j*-th industry $Q_j$. Note that there is a one-to-one mapping from the *i* commodities to the *j* industries as all industries produce only one product. The right-hand side of (13) represents the *j*-th industry's costs comprising intermediate input costs $\sum_{k \in COM} \sum_{s \in SRC} P_{ksj}^{INT} Q_{ksj}^{INT}$ and primary factor costs $\sum_{f \in FAC} P_{fj}^{FAC} Q_{fj}^{FAC}$ .

Equation (13) requires that industry output adjust so that the left-hand side (industry revenue) is always equal to the right-hand side (industry costs) thus ensuring that an industry's revenue is always exhausted on the cost of its inputs. This requires that $P_i^{BAS}$ is linked to $Q_j$. This is accomplished by a market-clearing condition.

Output prices are determined by a market-clearing condition for each commodity (i.e., total sales to all users equals output):

$$Q_j = \sum_{u \in ALLUSR} Q_{iu} + \sum_{u \in ALLUSR} \sum_{m \in MAR} Q_{ium}^{MAR} \qquad j \in IND, i \in COM. \qquad (14)$$

The left-hand side of (14) is output for the *j*-th industry. The right-hand side of (14) is the sum of non-margin sales to all users $\sum_{u \in ALLUSR} Q_{iu}$ and margin sales to all users $\sum_{u \in ALLUSR} \sum_{m \in MAR} Q_{ium}^{MAR}$ . Note that the set *ALLUSR* includes the set *USR* and exports sales and changes in stocks.

If demand for the *i*-th commodity rises at the initial output level, $P_i^{BAS}$ will rise. A rise in $P_i^{BAS}$ will increase revenue for the *j*-th industry via equation (13). At initial input quantities and prices this would normally lead to pure profits (i.e., revenues exceeding costs). But this is prevented by (13), which will cause output to rise thus driving up input quantities and prices until equality between revenue and costs is restored.

In a simple general equilibrium model, there are typically only two agents: households and firms. If the model represents a private ownership economy households will own all factors of production and thus firms, and profits by firms are transferred to households as income. The link between firm profits and household income determines that a general equilibrium exists (Starr, 1997). In a complex general equilibrium model with many agents as described here, factors of production are owned by households, foreigners and the governments. Despite this added complexity primary factor returns are assumed to accrue to the factor owner. This maintains the link between income for all agents and expenditure by all agents in each regional economy. This link determines the existence of a general equilibrium in the model described above.

## 7. Supply of factors of production

### 7.1. Land

Within each region we distinguish two types of land: primary production and non-primary production land. Primary production land is used only by the agricultural and mining industries. Non-primary production land consists of commercial land and residential land. Non-primary production land used by the dwellings sector represents residential land; non-primary production land used by all other sectors represents commercial land. There is a fixed supply of each type of land. For a given supply of each land type intersectoral movements are governed by a less restrictive version of the CET known as CRETH (constant ratio of elasticities of transformation, homothetic) function. A summary of the properties of CRETH functions and an illustration of their use in commodity supply analysis is given in Vincent *et al*. (1980).

Thus in each region the optimal supply of land is determined by the maximisation of after-tax land rentals subject to CRETH technology:

$$\sum_n \left[\frac{X_{nj}^{LND}}{X_n^{LND}}\right]^{b_i} \frac{W_n}{b_n} = \omega, \qquad b_n > 1, W_n > 0, \sum_n W_n = 1, \omega > 0,$$

$$n \in LND, j \in IND. \qquad (15)$$

In (15) $X_{nj}^{LND}$ is the supply of land type *n* to industry *j* and $X_n^{LND}$ is total supply of land of type *n*. Note that the prices applied in maximising (15) are after income taxes have been applied as the allocation of land is made by the owner of land not the user (i.e., the industry). The elasticity of substitution across occupations is $\sigma_i \sigma_j \big/ \sum_{n \in LND} S_n \sigma_n$ where $\sigma_i = 1/1 - b_i$ is the CRETH parameter associated with land type *n* and $S_n$ is the n-*th* land type's revenue share. The elasticity of transformation is set to -0.1 for primary production land making it relatively immobile across primary industries, and to -0.2 for land used by the non-dwellings sectors, and to -0.1 for land used by the dwellings sector. This means that non-primary production land is more mobile across the non-dwellings sectors than it is across the dwellings and non-dwellings sectors. The underlying assumption is that non-primary production land cannot be easily transferred between commercial and residential uses. For each type of land there is an industry-specific rental price that is determined by a market-clearing condition.

## 7.2. Labour

In each region there is an infinitely-lived representative household that decides on the supply of each of the *o* (=1,..,9) labour types $X_o^{LAB}$ based on a labour-leisure tradeoff that allows workers in each occupation to respond to changes in the real after-income-tax wage rate $\left(\frac{PWAGE_o}{CPI}\right)$, thus determining the hours of work they offer to the labour market. The labour-leisure tradeoff recognises the disutility of work. This gives upward-sloping labour supply curves for occupations as

$$\frac{X_o^{LAB}}{POP} = \left(\frac{PWAGE_o}{CPI}\right)^{\sigma_o^{LAB}} \quad , \quad o \in OCC, \tag{16}$$

where POP is population, CPI is the consumer price index and $\sigma_o^{LAB}$ is the uncompensated labour supply elasticity. The elasticity of labour supply is set at 0.15 reflecting econometric evidence on labour supply in the UK (Bargain *et al.*, 2011). For each occupation there is an occupation-specific wage rate that is determined by a market-clearing condition.

Unlike the supply of occupations, the supply of owner-operator labour in each region is determined at the industry level recognising that the return to such labour varies by industry depending on many factors. Thus, the supply of owner-operator labour by industry *j* $X_j^{OWN}$ is a positive function of population and the CPI-deflated real after-tax rental rate on owner-operator labour in industry *j* $POWN_j$:

$$\frac{X_j^{OWN}}{POP} = \left(\frac{POWN_j}{CPI}\right)^{\sigma^{OWN}} \quad , \quad j \in IND. \tag{17}$$

The supply elasticity is defined as $\sigma^{OWN} = \sum_{o=1}^{9} \sigma_o^{LAB} \big/ 9$. The rental rate on owner-operator labour is defined as the average of the rental rate on all non-labour factors of production. Note that the treatment applied in (17) combined with the definition of $\sigma^{OWN}$ ensures that owner-operator labour has a similar supply elasticity as regular labour recognising that the wage and rental rates of the two labour types vary.

Note that in the above treatment of labour supply decisions are made by an infinitely-lived representative household. A limitation of this approach is that labour supply responses will not reflect the heterogeneity of preferences to supply labour across households. In tax policy analysis, this treatment will underestimate the marginal excess burden of the personal income tax system. This limitation is muted somewhat as labour supply responses can vary by occupation through occupation-specific wage rates and, thus, there will be some heterogeneity in labour supply responses across occupations depending on how relative wage rates respond to a tax policy change.

## 7.3. Capital

Each industry uses capital specific to its own production process. Thus, the supply of capital is specified separately for each industry within each region. $K_{j,t}$ An industry's capital stock available for use in year *t* equals its capital at the start of year *t*-1 $K_{j,t-1}$ less any capital depreciation during year *t*-1 $\delta_j K_{j,t-1}$ plus any capital created (i.e., investment) during year *t*-1 $Q_{j,t-1}^{INV}$:[7]

[7] The determination of capital creation in each year is explained in section 9.1.

$$K_{jr,t} = \left(1-\delta_j\right) K_{jr,t-1} + Q^{INV}_{jr,t-1}, \quad j \in IND,\ \forall t. \tag{18}$$

Note $\delta_j$ is the constant rate of depreciation per period; thus, capital is assumed to depreciate geometrically over time. The representation of capital accumulation in equation (18) assumes that there is a one year gestation lag between investment by firms and an increment to the capital available for use by firms. For each type of capital there is an industry-specific rental rate that is determined by a market-clearing condition.

## 8. Factor prices paid by industry

As described in the previous section, the supply of primary factors to industries is determined by the price received by the owner of the factor. This price usually differs from the price paid by an industry for the factor. The difference between the two prices is attributable to factor income taxes and industry-specific factor taxes, such as land and labour taxes. For the UK, council tax is an example of land tax and employers' national insurance contributions are an example of a labour tax.

The price paid by an industry for a factor is defined as

$$P^{FACIND}_{if} = P^{FAC}_{if}\left(1+T^{FAC}_{if}\right) \quad i \in IND, f \in FAC, \tag{19}$$

where $P^{FACIND}_{if}$ is the price paid by industry *i* for factor *f*, $P^{FAC}_{if}$ is the pre-income-tax price received by owners of factor *f* used in industry *i*, and $T^{FAC}_{if}$ is the *ad valorem* rate of industry-specific tax on factor *f* used by industry *i*.

The price received by owners of a factor, net of income taxes, is

$$P^{FACNET}_{if} = P^{FAC}_{if}\left(1-T^{INC}_{if}\right) \quad i \in IND, f \in FAC, \tag{20}$$

where $P^{FACNET}_{if}$ is the post-income-tax price received by owners of factor *f* used in industry *i*, $T^{INC}_{if}$ is the *ad valorem* income tax rate on factor *f* used by industry *i*.

The different forms of the tax terms in equations (19) and (20) are attributable to the income tax rate $T^{INC}$ being defined as a rate relative to *gross* income rather than as a rate relative to a net-of-tax value as is the case for the industry-specific factor tax rate .

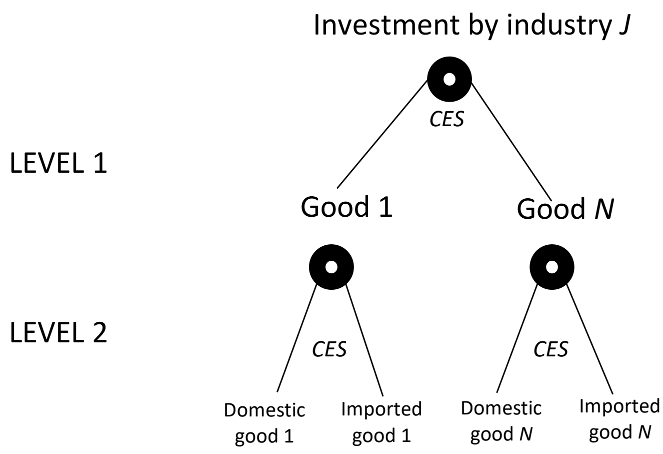
The provision of some examples helps tie down the more general notation of the current section to the factor-specific notation of the previous section:

- If factor *f* is land of type *n* then $P^{FACNET}_{if} = P^{LND}_{in}$; and
- If factor *f* is labour of occupation *o* then $P^{FACNET}_{if} = PWAGE_o,\ \forall i \in IND.$

## 9. Input technology: investment

Capital is assumed to be specific to each industry within a region. Consistent with this investment (or capital creation) is also specific to each industry. As apparent from Figure 2 the creation of investment (or capital goods) for each industry is determined in a two-tiered hierarchical structure. Given a level of investment by industry, capital creators first determine composite inputs to investment (level 1) and then determine inputs to investment by source (level 2) using CES technology in both cases.

**Figure 2. Input technology for investment (capital creation)**




## 9.1. Investment by industry

In each region investment in each industry is determined as a positive function of the post-income-tax, net-of-depreciation rate of return on the industry's capital, $ROR_j$ :

$$ROR_j = \frac{P_{jf}^{FACNET} K_j - \delta_j P_j^{INV} K_j}{P_j^{INV} K_j}, \quad i \in IND,\ f{=}capital, \tag{21}$$

where $P_j^{INV}$ is the purchasers' price of investment for industry $j$ . Note that all variables in equation (21) are contemporaneous. Equation (21) defines $ROR_j$ as post-income-tax rentals on capital $P_{jf}^{FACNET} K_j$ (*f=capital*) less capital depreciation $\delta_j P_j^{INV} K_j$ divided by the replacement cost of capital, $P_j^{INV} K_j$. The definition of $ROR_j$ is equivalent to Tobin's Q adjusted for taxes and depreciation.

During a simulation $ROR_j$ is able to fluctuate (i.e., it is endogenous) in the shortrun but will return to its initial value in the longrun. This is achieved by making investment $Q_j^{INV}$ in year *t* a positive function of $ROR_{jr}$ in year *t*:

$$1 + \frac{Q_{jt}^{INV} - \delta_j K_{jt}}{K_{jt}} = \left[1 + ROR_{jt}\right]^{\gamma} F_{jt}^{INV}, \quad i \in IND,\ \forall t, \tag{22}$$

where $\gamma$ is the elasticity of the capital growth rate with respect to the rate of return, and $F_j^{INV}$ is a positive constant. Equation (22) is written using transformed formed versions (i.e., by adding one) of the proportionate growth in industry *j*'s capital stock $\left(1 + \frac{Q_{jt}^{INV} - \delta_j K_{jt}}{K_{jt}}\right)$ and the rate of return $\left(1 + ROR_{jt}\right)$. That is, both are specified so that if either the rate of return or the proportionate growth in the capital stock pass through zero there will be no computational problems. With $\gamma$ = 2, a higher rate of return will lead to higher investment and higher proportionate growth in an industry's capital stock.

## 9.2. Level 1: Composite inputs to investment

At level one, the capital creator determines the cost-minimising mix of effective composite inputs to capital creation $Q_j^{INV}$ subject to CES production technology

$$Q_j^{INV} = \left[ \sum\nolimits_i \xi_i \left( \frac{Q_{ij}^{INV}}{A_{ij}^{INV}} \right)^{-\kappa} \right]^{-1/\kappa}, \quad 0 < \xi_i < 1, \sum_i \xi_i = 1, \kappa \geq -1, \kappa \neq 0,$$

$$i \in COM, j \in IND. \qquad (23)$$

In (23) $Q_{ij}^{INV}$ is commodity composite $i$ used by industry $j$, $A_{ij}^{INV}$ are unit input requirements, and $\xi_i$ and $\kappa$ are parameters. The CES elasticity of substitution is $\sigma = 1/(1+\kappa)$ = 0.1. This makes inputs to capital creation close to fixed shares of industry investment levels and relatively unresponsive to changes in relative prices.

## 9.3. Level 2: Domestic and imported inputs to investment

At the second level of the hierarchical structure capital creators in industry $j$ choose the optimal mix of domestic and foreign inputs to minimise the costs of producing units of capital subject to CES technology

$$Q_{ij}^{INV} = \left[ \sum\nolimits_s \varsigma_s \left( \frac{Q_{isj}^{INV}}{A_{isj}^{INV}} \right)^{-\psi_i} \right]^{-1/\psi_i}, \quad 0 < \varsigma_s < 1, \sum_s \varsigma_s = 1, \psi_i \geq -1, \psi_i \neq 0,$$

$$i \in COM, s \in SRC, j \in IND. \qquad (24)$$

Thus capital in each industry is produced with inputs of domestically-produced $\left(Q_{isjr}^{INV}, s = DOM\right)$ and imported commodities $\left(Q_{isjr}^{INV}, s = IMP\right)$. No primary factors are used directly as inputs to capital formation. Nevertheless, primary factors are used in the production of the commodity inputs to investment. The CES elasticity of substitution for the $i$-th input is $\sigma_i = 1/(1+\psi_i)$. These values are drawn from econometric estimates based on UK data over the period 1998 to 2016. This work is described in section 0. The parameter values allow input demands to be responsive to relative price changes between domestic and foreign goods.

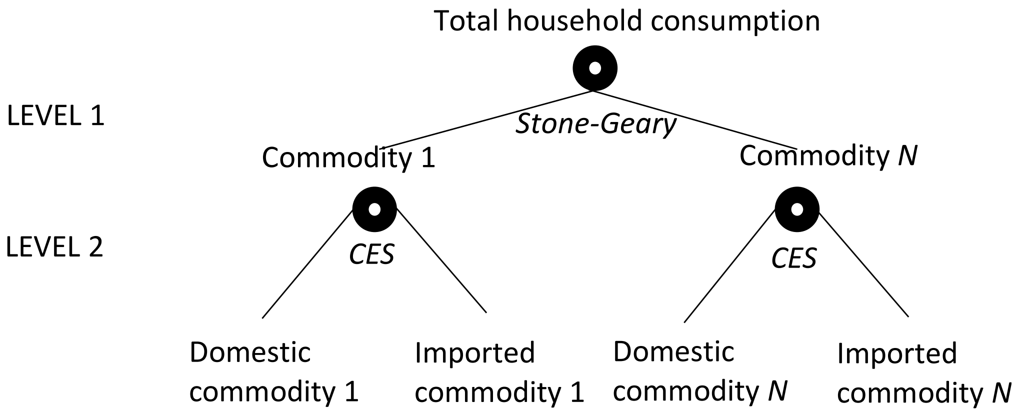
## 10. Household demands

The most common functional form in consumer theory is the Cobb-Douglas utility function, which displays constant average budget shares. Values for the price and income elasticities from maximisation of the Cobb-Douglas utility function equal unity. This is recognised as a drawback since unitary uncompensated own-price and income elasticities are not consistent with empirical evidence. Therefore, using the Cobb-Douglas functional form can give rise to biased estimates of behaviour for many general equilibrium simulations (Hertel and Tsigas, 1997).

Given the restrictive assumptions of Cobb-Douglas preferences, the CES utility function has become a popular functional form in the calibration process of CGE models. The CES function relaxes some of the assumptions of the Cobb-Douglas utility function by requiring that price elasticities are estimated rather than assumed; thus the CES's major strength is that it allows for the possibility of non-unitary price elasticities. Regardless, CES values for income elasticities still equal one. Theoretically, unitary income elasticities imply consumer preferences are homothetic in income, i.e., that budget shares for each commodity are entirely independent of the level of income. Homothetic preferences are unsupported by empirical work (Clements *et al.*, 1995). This limitation can be overcome by using a Stone-Geary (Geary, 1950; Stone, 1954) or Klein-Rubin (Klein and Rubin, 1948) utility function to represent consumer preferences.

Here we assume that each region has an infinitely-lived representative household that maximises nested utility functions subject to a budget constraint (see Figure 3). At the first level the representative household maximises a Stone-Geary utility function by consuming combinations of composite commodities. At the second level the representative household determines the optimal mix of domestic and imported varieties that combine to form composite commodities using CES technology.

**Figure 3. Input technology for households (utility)**




## 10.1. Level 1: Composite inputs to household consumption

The regional representative household determines the optimal mix of composite commodities by maximising a Stone-Geary utility function

$$U^H = \prod_i \left(Q_i^H - QSUB_i^H\right)^{\beta_i}, \quad 0 < \beta_i < 1, \ \sum_i \beta_i = 1, \quad i \in COM \qquad (25)$$

where $Q_i^H$ and $QSUB_i^H$ are total household demand and subsistence household demand for the $i$-th commodity composite. A further constraint in (25) is $Q_i^H > QSUB_i^H$. With Stone-Geary utility the consumer first allocates an amount of income to the subsistence quantities; theses are purchased regardless of price and income.

Maximisation of (25) subject to the income constraint $M = \sum_{i=1}^{n} P_i^H Q_i^H$, where $M$ is total income (or expenditure) and $P_i^H$ is the consumer price for the $i$-th good, yields the linear expenditure system (LES) Marshallian demand function

$$Q_i^H = QSUB_i^H + \frac{\beta_i}{P_i^H}\left(M - \sum_j P_j^H QSUB_j^H\right), \quad i, j \in COM. \qquad (26)$$

The name LES derives from the property that expenditure on each good is a linear function of prices and income (expenditure). The term in parentheses $\left(M - \sum_j P_j^H QSUB_j^H\right)$ refers to supernumerary (or luxury) income $\left(WLUX^H\right)$, representing the income available after the consumption of the subsistence bundle has been allocated. Thus, the LES divides total consumption of the $i$-th commodity composite into two components: a subsistence (or minimum) part $QSUB_i^H$ and a luxury (or supernumerary) part $\frac{\beta_i}{P_i^H}\left(M-\sum_j P_j^H QSUB_j^H\right)$.

Note that with $QSUB_i^H$ constant $\beta_i$ represents the marginal budget share $\frac{\partial\left(P_i^H Q_i^H\right)}{\partial M}$, i.e., the change in expenditure on good $i$ from a one-dollar change in income. Let $w_i=\frac{P_i^H Q_i^H}{M}$ represent the budget share for the $i$-th commodity. Then, $\eta_i=\frac{\beta_i}{w_i}$ is the $i$-th income elasticity with the constraint that $\sum_i w_i\eta_i=1$. Clements *et al*. (2020) show that the ($i$,$j$)-th Marshallian price elasticity $\eta_{ij}^*=\delta_{ij}\left(\frac{s_i}{w_i}-1\right)-\frac{\beta_i}{w_i}s_j$ where $\delta_{ij}$ is the Kronecker delta and $s_i=\frac{P_i^H QSUB_i^H}{M}$, i.e., the subsistence budget share of good $i$.

The above definitions of the income and price elasticities show the importance of the marginal budget shares $\beta_i$ in determining the properties of the LES. The $\beta_i$ parameters are estimated from UK data over the period 1998-2017. To determine income elasticities $\eta_i$ we apply $\beta_i=-\frac{\eta_i}{\omega}$ where $\omega$ is the 'Frisch' parameter - the income elasticity of the marginal utility of income (Theil, 1980). The value of $\omega$ is based on a range of estimates for the UK. The sources and methods for estimating $\beta_i$ and $\eta_i$ are described in section 18.

### 10.2. Level 2: Domestic and imported inputs to household consumption

At the second level of the utility nest household demand is characterised by the CES aggregation of domestically-produced goods $Q_{is}^H$ ($s$=*DOM*) and imports $Q_{is}^H$ ($s$=*IMP*) that are considered imperfect substitutes

$$Q_i^H=\left[\sum_s S_{is}\left(Q_{is}^H\right)^{-\zeta_i}\right]^{-1/\zeta_i},\quad 0<S_{is}<1,\sum_s S_{is}=1,\zeta_i\geq -1,\zeta_i\neq 0,$$

$$i\in COM,\ s\in SRC, \tag{27}$$

where $S_{is}$ and $\zeta_i$ are parameters. The CES elasticity of substitution for the $i$-th composite is $\sigma_i=1/(1+\zeta_i)$. These values are drawn from econometric estimates based on UK data over the period 1998 to 2016. This work is described in section 0.

## 11. Export demands

Export demands by foreigners are treated differently for tourism and non-tourism commodities. In each region export demands for non-tourism commodities (represented by the set *NONTOUR*) to destination region $d$ (= EU, non-EU) are determined by a constant elasticity of demand function

$$Q_{id}^{EXP}=F_{id}^{EXP}\cdot F^{EXP}\left(PFC_{id}^{EXP}\right)^{-\vartheta_i},\quad \vartheta>0,\quad i\in NONTOUR,\ d\in DEST \tag{28}$$

where $Q_{id}^{EXP}$ is exports of commodity $i$ to destination $d$, and $F_{id}^{EXP}$ and $F^{EXP}$ represent shifts in commodity and regional exports, and $PFC_{id}^{EXP}$ is the foreign currency price of exports of commodity *i* to region *d*. $\vartheta_i$ is the elasticity of demand for commodity *i*. It is assumed that the UK has little market power in its export markets, and so $\vartheta_i$ is set to 12 for all non-tourism commodities.

In each region export demands for tourism commodities (represented by the set *TOUR*) are treated as a bundle. The bundle represents purchases made by foreign tourists to the UK and includes expenditure on accommodation, restaurants, transport, the arts, recreation services, etc.[8] The tourism bundle is determined by a constant elasticity of demand function

$$Q_{id}^{EXP} = F_{id}^{EXP} \cdot F^{EXP} \left( PTOUR_d^{EXP} \right)^{-\varpi_i}, \qquad i \in TOUR,\, d \in DEST, \tag{29}$$

where $PTOUR_d^{EXP} = \sum_{i \in TOUR} S_i^{EXP} PFC_{id}^{EXP}$ , i.e., the price of the tourism bundle faced by consumers in region *d*. $\varpi_i$ is set to 10 for all tourism commodities. This treatment of tourism commodities makes export demand very elastic for the tourism bundle, i.e., the UK has little market power as a tourism destination, but foreigners purchase units of these commodities in a fixed pattern.

The foreign currency price of exports is defined as

$$PFC_i^{EXP} = PFOB_i^{EXP} \cdot E\,, \qquad i \in COM, \tag{30}$$

where $PFOB_i^{EXP}$ is the FOB[9] domestic currency price of exports and *E* is the exchange rate defined as foreign currency price of a unit of domestic currency.

The FOB domestic currency price of exports is defined as

$$PFOB_i^{EXP} = P_{iu}^{PREVAT} \left( 1 + T_{iu}^{VAT} \right). \qquad i \in COM,\, u{=}EXP. \tag{31}$$

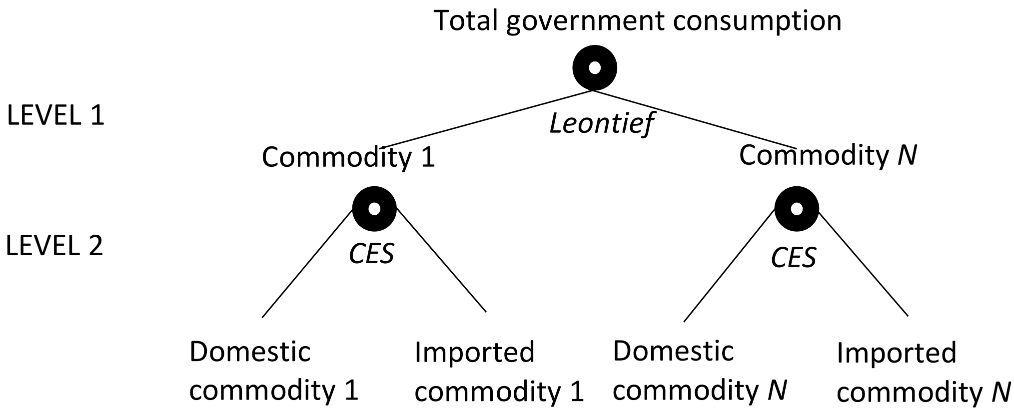
In equation (31) $P_{iu}^{PREVAT}$ represents the pre-VAT price of exports and $T_{iur}^{VAT}$ is the *ad valorem* VAT rate applied to commodity *i* for export. The pre-VAT price of exports is defined similarly to the pre-VAT price of domestic goods, as described in section 4.

## 12. Government consumption and expenditure

We represent governments as affecting the economy by purchasing goods and services, collecting taxes, receiving revenue from government-owned assets, and making transfer payments. Within each region both the central and the local government of the region demand commodities from each region. While the shares of each commodity in government demand may vary across regions and between governments, there is a consistent structure of demand for all government consumption activities as shown in Figure 4. As apparent from the figure, at level 1 the government sector determines composite inputs by applying a Leontief utility function. At level 2, the government sector chooses an optimal mix of domestically-produced and imported and goods assuming CES preferences.

[8] A complete listing of tourism commodities appears in Table 2.
[9] Free On Board.

**Figure 4. Input technology for governments**




### 12.1. Level 1: Composite inputs to goverment consumption

In each region the *i* composite inputs to consumption by government *g* $Q^{G}_{gi}$ are a Leontief function of total government consumption $Q^{G}_{\bullet}$

$$Q^{ACT}_{gi} = \min\left[Q^{G}_{g}\right]\cdot\left(F^{G}_{g}\right), \quad g \in GOV,\ i \in COM. \tag{32}$$

The set *GOV* comprises the central ($g = C$) and local ($g = L$) government. $F^{G}_{g}$ is a shift term that allows for specific targeting of government demands in aggregate. For instance, in most simulations $Q^{G}_{g}$ is exogenous in order to enforce zero change in total government demands, or a given change in total government demands.

### 12.2. Level 2: Domestic and imported inputs to goverment consumption

In each region the optimal combination of domestic and imported inputs to the *i* composite inputs to government consumption $Q^{G}_{gis}$ ($s \in SRC$) are determined via CES technology. Nevertheless, the 2013 UK input-output tables show no direct government consumption of imported commodities. Thus, $Q^{G}_{gis}$ = $Q^{G}_{gi}$ (*s*=*DOM*) and $Q^{G}_{gis}$ = 0 (*s*=*IMP*).

### 12.3. Government transfer payments

A range of transfer payments are represented including unemployment benefits, benefits to the aged and a composite of other government benefits payments. These payments are mostly undertaken by the central government. These transfers move with an appropriate volume base (e.g., the number of unemployed persons, the aged population or population). The price component of central government benefits is indexed to the national average nominal wage rate; the price component of local government benefits is indexed to the GRP (gross regional product) price index within the region in which benefits are paid. Interest payments on government debt are made to households and these form part of government transfer payments. Interest payments are calculated as the product of the interest rate on government debt and the level of the debt.

The budget balance of all governments is typically exogenous (relative to GDP) in baseline and policy simulations. This is accommodated by allowing a variable that would usually be exogenous, such as a tax rate, to vary. The usual tax rate is that applying to personal income.

### 12.4. Other government expenditure

For each level of government a range of other government expenditures are represented including other operating expenses, government investment expenditure and capital expenditure on existing assets. Other operating expenses are linked to aggregate government consumption. Government investment expenditure is determined as the product of the government investment demands by industry and the industry-specific investment price index. Government investment demands are typically held exogenous or imposed. Capital expenditure on existing assets move with GDP or GRP.

## 13. Inter-regional trade

### 13.1. Theory

Two equations specify the pattern of trade across regions. The equations are both CES demand functions for each commodity, with total domestic commodity use within a region being a CES composite of imports from all regions. There is an equation for non-margin use and margin use of commodities.[10]

Non-margin trade is represented as

$$Q_{cs}^{DUSE} = \left[ \sum_r \mathrm{B}_{crs} \left( Q_{crs}^{IRT} \right)^{-\iota} \right]^{\frac{-1}{\iota}} \qquad 0 < \mathrm{B}_{crs} < 1, \sum_r \mathrm{B}_{crs} = 1, \iota_c \geq -1, \iota_c \neq 0, \quad c \in COM,\ s,r \in REG, \tag{33}$$

where $Q_{crs}^{IRT}$ is the quantity of commodity *c* traded from source region *r* to destination region *s* and $Q_{cs}^{DUSE}$ is the quantity of domestic commodity *c* used in region *s*. $\mathrm{B}_{crs}$ and $\iota_c$ are parameters. The CES elasticity of substitution for the *c*-th commodity is $\sigma_c = 1/(1+\iota_c)$. These elasticities have a wide range of values – from 1 to 7.5 – according to the degree of substitutability between domestic sources of each commodity. For each primary and secondary good, the interregional trade elasticity is double the good's international trade elasticities, that is, the elasticities of substitution between international imports of the good and the domestic good (see section 0). This approach to assigning values of $\sigma_c$ is also applied for internationally-traded services, for example, air and water transport, and embodies the idea that these goods and services are more easily traded across internal borders than across international borders. For all other services, the interregional trade elasticities are set to 1, so that for these services each region will purchase fixed shares, in value terms, from each other region.

The quantity and basic price of a domestic commodity used in a particular region – variables $P_{cs}^{BAS}$ and $Q_{cs}^{DUSE}$ – are CES composites of the prices and quantities of the domestically-produced commodity imported from each region. The price of a domestically produced commodity sold to another region is the price of the local commodity produced in the source region, that is:

$$P_{crs}^{IRT} = P_{cr}^{LOC} \qquad c \in COM,\ s,r \in REG. \tag{34}$$

*Intra*-regional trade, where the source and destination regions in equation (33) are the same, is not zero, but is usually the bulk of trade across regions in most commodities. The entire amount of each local commodity produced in a particular region *q* is (except for any

[10] There are seven margin commodities: (i) electricity, transmission and distribution; (ii) gas; distribution of gaseous fuels through mains; steam and air conditioning supply; (iii) wholesale and retail trade and repair services of motor vehicles and motorcycles; (iv) wholesale trade services, except of motor vehicles and motorcycles; (v) rail transport services; (vi) land transport services and transport services via pipelines, excluding rail transport; and (vii) water transport services.

change in inventories) either sold to another region ($r$=$v$, $s$≠$v$ in equation (33)) or sold to the region of production ($r$=$s$=$v$ in equation (33)), where it is then combined with true *inter-regional* imports from other regions ($r$≠$v$, $s$=$v$ in equation (33)) to form the domestic commodity used in region $q$.

This approach utilises a common sourcing assumption with respect to the use of domestic commodities. Activities in a region – production, investment and government and household consumption – each use some portion of each domestic commodity available for use in the region, that is, some portion of the CES composite of intra- and inter-regional trade represented by the variable $Q^{DUSE}$. Therefore, the share of a particular producing region $r$ in the use of a domestic commodity in region $s$ is the same across all activities using the commodity, being equal to the share of region $r$ in the CES composite $Q^{DUSE}$. Common sourcing assumptions are also used in the TERM model (Horridge, 2011) and the multi-country, multi-commodity CGE model GTAP (Hertel and Tsigas, 1997). Common sourcing assumptions reduce considerably the size and computational burden of solving large systems of simultaneous equations.

Inter-regional margin trade is represented similarly to non-margin as in equation (33). The CES elasticity of substitution is 0.2 for all margin commodities representing a relatively low degree of substitution, that is, little opportunity for swapping margin suppliers in the UK.

The price of a domestically-produced commodity sold to another region for margin use is the price of the local commodity produced in the source region, that is:

$$P^{MARIRT}_{mrs} = P^{LOC}_{mr} \qquad m \in MAR,\ s,r \in REG. \tag{35}$$

The prices of the margin and non-margin components of a local commodity produced in a particular source region are equal. Nevertheless, the prices of non-margin use and margin use of a domestic commodity within a destination region – variables $P^{BAS}$ and $P^{MAR}$ – will usually differ. The weight of a particular source region in forming the composite price $P^{BAS}$ will not necessarily be the same as the weight of that region in forming the composite price $P^{MAR}$. The discussion of inter-regional trade data in the next section will explain how these weights arise and why they usually differ.

### 13.2. The derivation of inter-regional trade data

Inter-regional trade flows are determined by the requirement that total sales equal total costs of each regional industry, that is, that the regional IO tables are balanced. Shares of each region in the national output of an industry are used to split each of the input costs of the industry in the national IO table across regions. These shares are typically based on employment or labour hours shares (see section 17). That is, each input cost for a particular regional industry is equal to the industry-specific share times the corresponding input cost in the national IO table. So total costs for the particular regional industry is equal to the industry-specific share times the total costs of the national industry. Nevertheless, regional sales of the commodity associated with a regional industry are derived by multiplying all uses of the commodity in the national IO table by a range of different industry-specific shares depending on what industry or other activity is using the commodity. Consequently, total sales of the commodity calculated from the regional IO table are highly unlikely to equal total costs, and inter-regional trade flows are calculated to offset the differences.

For commodities that have a margin use, the ratio of margin output to total output of the producing industry is used to determine how much of the rebalancing inter-regional trade flow should be allocated to non-margin versus margin trade. The ratio of margin output to total output for each industry is assumed to be constant across regions and therefore equal to the national value of the ratio. Margin output for a particular regional industry is calculated by multiplying the ratio by the total costs of the industry. Regional margin usage will be represented by the summation of the relevant rows in the regional IO table. The difference determines the offsetting inter-regional margin trade that must be introduced to the database. A similar approach is used for non-margin usage. If the non-margin and margin components of a commodity vary across activities in their patterns of use then the inter-regional trade patterns of the two components could differ, as alluded to at the conclusion of the previous section.

## 14. Asset and liability accumulation

### 14.1. Foreign assets and liabilities

The model specifies foreign assets and liabilities held by the representative household. Foreign assets comprise equity $\left(FE_r\right)$ and credit $\left(FC_r\right)$ instruments; foreign liabilities comprise foreign direct investment $\left(FDI_r\right)$ and debt $\left(FD_r\right)$ instruments. Foreign credit can be issued in domestic currency $\left(FC_r^{DC}\right)$ and foreign currency $\left(FC_r^{FC}\right)$, and similarly for foreign debt $\left(FD_r^{DC}, FD_r^{FC}\right)$, which means that revaluation effects caused by changes in prices and the exchange rate will influence the accumulation of net foreign liabilities. This is an important mechanism as changes in net foreign liabilities have real effects. We can write foreign assets $\left(FA_r\right)$ and foreign liabilities $\left(FL_r\right)$ in any year as

$$FA_r = FE_r + FC_r^{DC} + FC_r^{FC} \qquad r \in REG. \tag{36}$$

$$FL_r = FDI_r + FD_r^{DC} + FD_r^{FC} \qquad r \in REG. \tag{37}$$

It is helpful in the following discussion to introduce a time subscript. All gross foreign asset variables $\left(FA_{r,t},\ FE_{r,t},\ FC_{r,t}^{DC},\ FC_{r,t}^{FC}\right)$ and gross foreign liability variables $\left(FL_{r,t},\ FDI_{r,t},\ FD_{r,t}^{DC},\ FD_{r,t}^{FC}\right)$ are defined as averages of stocks of across year *t*. Nevertheless, the net foreign liability variable $\left(NFL_{r,t}\right)$ is defined as the value of net foreign liabilities at the beginning of year *t*. The change in net foreign liabilities from the beginning of year *t* to the beginning of year *t*+1 $\left(\Delta NFL_{r,t}\right)$ will equal minus the current account from year *t*:

$$\Delta NFL_{r,t} = NFL_{r,t+1} - NFL_{r,t} = -CA_{r,t}\,. \tag{38}$$

To link net foreign liabilities to foreign assets and liabilities we assume that net foreign liabilities grow linearly across each year, that is:

$$NFL_{r,t+\tau} = NFL_{r,t} - \tau CA_{r,t}\,. \quad 0 \le \tau \le 1 \tag{39}$$

Therefore, the average net foreign liabilities across year *t* $\left(NFL_{r,t}^{AVE}\right)$ are:

$$\begin{aligned} NFL_{r,t}^{AVE} &= \int_0^1 NFL_{r,t+\tau}\, d\tau \\ &= NFL_{r,t} - 0.5CA_{r,t} \end{aligned} \tag{40}$$

The average net foreign liabilities can be expressed as the difference of the foreign asset and liability variables (all these variables being average stocks across a year) as:

$$NFL_{r,t}^{AVE} = FL_{r,t} - FA_{r,t}\,. \tag{41}$$

The substitution of equation (40) into equation (41) yields, after rearrangement of terms:

$$NFL_{r,t} = FL_{r,t} - FA_{r,t} + 0.5CA_{r,t}\,. \tag{42}$$

The current account is defined as the value of net exports (exports minus imports) plus the value of net foreign income. Net foreign income is the sum of income earned on foreign assets $\left(FE_r + FC_r^{DC} + FC_r^{FC}\right)$ minus income paid on foreign liabilities $\left(FDI_r + FD_r^{DC} + FD_r^{FC}\right)$. In a typical baseline simulation foreign equity and foreign credit will grow at the same rate as nominal GDP. Foreign debt will grow as the same rate as foreign direct investment. Foreign direct investment is determined as a residual that allows equation (42) to be consistent with equation (38). This means that the composition of foreign assets and liabilities will be constant across time. Nevertheless, the driver of the accumulation of net foreign liabilities will be the accumulation of foreign direct investment. This is appropriate as relative to other components of net foreign liabilities, foreign direct investment is the only component that is determined based on optimising behaviour. All other components of net foreign liabilities are assumed to have a fixed rate of return. Given all of these assumptions, equation (42) ensures foreign direct investment changes so that the net capital inflow (i.e., the balance on the capital account) is consistent with the balance on the current account.

In baseline and policy simulations the time path of net foreign liabilities relative to GDP can be treated in a range of ways. For example, the ratio can grow over time without stabilising in the final year, the ratio can grow over time but stabilise in the final year, or the ratio can be stable over time. Regardless of these choices, choosing a path for net foreign liabilities is achieved via an endogenous household saving rate that trades off household consumption and exports. For example, if the growth in net foreign liabilities must be slowed, this can be achieved by raising the saving rate. This will decrease the rate of growth in household consumption and increase the rate of growth in exports. This will improve the current account balance (i.e., reduce current account deficit or increase the surplus). An improvement in the current account balance will slow the rate of growth in net foreign liabilities.

### 14.2. Government debt

For local and central governments, debt at the beginning of year $t$+1 $\left(GD_{gr,t+1}\right)$ equals government debt at the beginning of year $t$ $\left(GD_{gr,t}\right)$ minus the government budget incurred during year $t$ $\left(GB_{gr,t}\right)$. This gives an accumulation equation similar to (38):

$$\Delta GD_{gr,t} = -GB_{gr,t-1} \qquad g = L,\ r \in REG,\ \forall t; \tag{43}$$

$$\Delta GD_{g,t} = -GB_{g,t-1} \qquad g = C,\ \forall t. \tag{44}$$

For all levels of government, the budget is defined as total revenue minus expenditure inclusive of interest on government debt.

Similar to net foreign liabilities, in baseline and policy simulations the time path of government debt relative to GDP can be treated in a range of ways. Choosing a path for government debt is achieved by adjusting government saving (i.e., the budget balance) via changes in the rate of growth in tax revenues or government expenditure. Tax revenues will usually be adjusted by raising or lowering the personal income tax rate. Government expenditure will usually be adjusted by increasing or decreasing the rate of growth in government consumption expenditure.

## 15. Macroeconomic closure

The model described here is dynamic. That is, the theory of the model refers to how model variables relate to each other within a period or across time periods. A dynamic simulation of the effects of a policy change involves running the model twice to generate the baseline and policy simulations. The baseline may be designed to be a plausible forecast of how the economy will evolve over time in the absence of the policy shock of interest. The baseline may incorporate some external forecasts for key macroeconomic variables in the short term with convergence to a balanced growth path in the long term. Alternatively, the baseline may represent the movement from a non-balanced to a balanced growth path. The balanced growth path is calibrated to reflect the 30-year annual growth rate in real GDP for the UK, i.e., 2.2%. This means that all quantity variables grow at this common rate. The balanced growth moves the economy from the initial steady-state to a new steady state. The new steady state is a point where the economy reaches a capital-labour ratio that can be sustained infinitely into the future.

Two aspects of the baseline apply budget constraints for the household and the government. Aggregate household consumption is determined by moving the ratio of the current account to GDP to a level that stabilises net foreign liabilities in the final year of the model time horizon (typically 30 years), the household saving rate adjusting to achieve this target. This ensures that in the longrun households consume at a rate that is sustainable relative to the growth in output. Similarly, the government budget as a ratio of GDP is slowly moved to a level that stabilises government debt in the final year of the model time horizon, the personal income tax rate adjusting to achieve this target. This ensures that in the longrun government spending is sustainable relative to the growth in output.

With the exception of the policy variables of interest (e.g., tax rates, technology, etc.), all exogenous variables in the policy simulation are assigned the values they had in the baseline simulation. The differences in the values of variables in the baseline and policy simulations quantify the effects of moving the variables of interest away from their baseline values, i.e., the deviations of variables from their baseline values caused by the policy shock modelled. The household and government budget constraints applied in the baseline also apply in the policy simulation. Note that the model time horizon must be long enough that the economy can reach a new steady-state given the policy shock of interest. For most shocks, 30 years is an adequate time horizon. Shocks that strongly perturb the rate of capital accumulation may require much longer time horizons to reach a new steady-state.

## 16. Model tests

Two types of tests are applied to check that the model has been properly specified and behaves as expected.

The first type of test checks for homogeneity with respect to prices and quantities in the model theory and data. Economic models, including general equilibrium models, have various homogeneity properties. Price homogeneity requires that all price variables are homogeneous of degree one in prices and homogeneous of degree zero in quantities. This means that one solution of the model is obtained by increasing all nominal variables (such as domestic prices, domestic dollar values and the exchange rate) by one per cent, while all volumes (such as physical quantities) remain unchanged. Quantity homogeneity requires that all quantity variables are homogeneous of degree one in quantities and homogeneous of degree zero in prices. This means that one solution of the model is obtained by increasing all quantity variables by one per cent, while all prices remain unchanged. Checking such homogeneity properties is one important way of verifying that the model has been implemented correctly.

The second type of test implements a realistic shock to the model that has a known solution. Typically this involves an exogenous increase in government consumption expenditures. Given the labour intensive nature of the majority of government consumption expenditures (e.g., education, health and public administration), in the shortrun such a shock will lead to increased employment in government-provided or government-dominated industries relative to other industries. With slowly-adjusting wage rates, it will also mean that employment will rise faster than labour supply. With no change in the capital stock this means a lower capital-labour ratio. GDP will increase and the current account will deteriorate mainly due to increased imports. As government consumption is generally untaxed or lightly taxed relative to other industries, the expansion in government consumption will increase the size of lightly-taxed industries (and commodities) and decrease the size of highly taxed industries (and commodities). Thus the tax base will contract. This reflects a reduction in allocative efficiency as the economy has moved further away from its optimal output level.

In the longrun this outcome is not sustainable. The movement of labour to government provided or funded activities will eventually bid up the real wage rate and employment will remain constant relative to labour supply. A higher real wage rate means that the real cost of labour is higher for all industries and the non-government industries will generally contract. For the reasons described earlier, the tax base will contract. The further contraction of the non-government industries will cause the capital stock to fall (i.e., a lower capital-labour ratio) and GDP will contract. Higher labour costs for domestic producers will mean lower exports and a further deterioration in the current account. If the current account is to return to sustainable levels, the household saving rate must rise to allow household consumption to fall and exports to rise. Thus increase in government consumption will be more than offset by a fall in household consumption.

Implementing an exogenous increase in government consumption and checking the model results against the behaviour described above is another important way of verifying that the model has been implemented correctly.

## 17. Data sources and methods

Here we describe the process of developing the data used to calibrate the model. Table 1 provides a brief description of each block of data and summarises the source. Table 2 lists the 127 commodities in the model database. As all commodities are produced by a single representative industry, this is also the number of industries in the model. The listing of commodities and industries comprises (i) the 105 commodities and industries in the product-by-industry tables provided by the ONS, (ii) 11 commodities and industries representing services provided by the government sector, and (iii) 11 commodities and industries representing services provided by not-for-profit institutions serving households. The industries represented align with the UK Standard Industrial Classification at the 2-digit level.[11] Table 3 presents a stylised description of the recursive process applied to covert raw data in a format compatible with the model theory.

[11] See https://onsdigital.github.io/dp-classification-tools/standard-industrial-classification/ONS_SIC_hierarchy_view.html.

### Table 1. Data description and sources

| Item | Description | Source |
|---|---|---|
| **Input-output (IO) table:** domestic use, basic prices, product by product | This is the core data of the model. The table is simplified to produce the 'make' or production matrix by assuming each product is produced by a single representative industry. | ONS IO analytical tables, reference number 007230. *https://www.ons.gov.uk/economy/nationalaccounts/supplyandusetables/adhocs/007230inputoutputanalyticaltables2013summary* |
| **Imports use table:** basic prices, product by product | This supplements the IO table by detailing the import values by product and user. | ONS IO analytical tables, reference number 007230. *https://www.ons.gov.uk/economy/nationalaccounts/supplyandusetables/adhocs/007230inputoutputanalyticaltables2013summary* |
| **Taxes and subsidies** on production and products | This data is provided as the sum of taxes (subsidies) on a product supplied to all users, and the sum of taxes (subsidies) on all production regardless of the user. The ONS Supply-Use table team have informed us that this data should be available in the near-term, and is awaiting approval to be released. We estimate as a temporary measure. | ONS website under user-requested data reference number 006297. *https://www.ons.gov.uk/economy/nationalaccounts/supplyandusetables/adhocs/006297taxesandsubsidesonproductsandproduction1997to2014* |
| **Margins** | Margins (along with taxes and subsidies) determine the difference between producer prices and user prices for products. These data appear in the IO table as 'Distributors' Trading Margins'. ONS does not include transport as part of margins, instead they are inputs to the wholesale and retail industries. A rich dataset on wholesale and retail trade is available in the Annual Business Survey which we use in estimating transport margins. | ONS website as 'Input-output supply and use tables' (no reference number). *https://www.ons.gov.uk/economy/nationalaccounts/supplyandusetables/datasets/inputoutputsupplyandusetables* |
| **Self-employment income** by industry | These data are used to separate the component of GOS that is paid directly to workers who are self-employed. These data were only available for the 19 major industry groups. A request was made to ONS and is now available for supply use table industries. | ONS website reference number 007583. *https://www.ons.gov.uk/economy/grossdomesticproductgdp/adhocs/007583breakdownofgrossoperatingsurplusmixedincomeoftheukbyindustrysection2007to2014* |
| **Capital stocks** by industry | Capital stock data represents initial quantities of capital used by firms. | ONS website at *https://www.ons.gov.uk/economy/nationalaccounts/uksectoraccounts/datasets/capitalstock* |

### Table 1. Data description and sources (cont.)

| Item | Description | Source |
|---|---|---|
| **Gross fixed capital formation (GFCF)** by industry and product | GCFC data is specifies the inputs to capital formation by industry. | ONS website under 'Input-output supply and use tables' (no reference number). *https://www.ons.gov.uk/economy/nationalaccounts/supplyandusetables/datasets/inputoutputsupplyandusetables* |
| **Employers' national insurance contributions** | This is a component of labour taxes paid by firms. The data is combined with employer contributions to pension funds as social security payments. Note that there are some excluded industries in this data set, which need to be approximated. | ONS website as part of the Annual Business Survey. *https://www.ons.gov.uk/businessindustryandtrade/business/businessservices/datasets/uknonfinancialbusinesseconomyannualbusinesssurveysections* |
| **Regional shares** (12 regions) | The annual businesses survey contains several metrics on industries across 12 regions (9 in England, Wales, Scotland and Northern Ireland). Note that some industries are not captured in the survey. | ONS website at *https://www.ons.gov.uk/businessindustryandtrade/business/businessservices/datasets/uknonfinancialbusinesseconomyannualbusinesssurveyregionalresultssectionsas* |
| **Employment** by region | A high level of employment by industry and region is available, which can be used for further regional disaggregation of industries. | Nomis website at *https://www.nomisweb.co.uk/query/construct/submit.asp?menuopt=201&subcomp* |

| | | |
|---|---|---|
| **Scotland IO tables** | This table is used either to determine shares, or verify if shares used are producing a good representation of the Scotland economy. | *http://www.gov.scot/Topics/Statistics/Browse/Economy/Input-Output/Downloads* |
| **Employment** by industry | Data on employment by industry at the 4-digit level was mapped to the SUT categories. These then provided row totals for a RAS process that gave a balanced matrix of employment by industry and occupation. | ONS website at *https://www.ons.gov.uk/employmentandlabourmarket/peopleinwork/earningsandworkinghours/datasets/industry4digitsic2007ashetable16* |
| **Occupation** by industry | The occupation by industry matrix is taken from the 2011 census. This data provides a relationship between occupations of workers and the industry. | Nomis website at *https://www.nomisweb.co.uk/census/2011/ct0144* |

**Note: The model data is specified in £ millions. Most data is for 2013 although some sources provide more recent data. There are several version of IO data files on the ONS website, the hyperlinks provided link to the exact source.**

Table 2. Commodities and industries

| ONS industry code | | Industry description |
|---|---|---|
| 1. | 01 | Products of agriculture, hunting and related services |
| 2. | 02 | Products of forestry, logging and related services |
| 3. | 03 | Fish and other fishing products; aquaculture products; support services to fishing |
| 4. | 05 | Coal and lignite |
| 5. | 06&07 | Extraction Of Crude Petroleum And Natural Gas & Mining Of Metal Ores |
| 6. | 08 | Other mining and quarrying products |
| 7. | 09 | Mining support services |
| 8. | 10.1 | Preserved meat and meat products |
| 9. | 10.2-3 | Processed and preserved fish, crustaceans, molluscs, fruit and vegetables |
| 10. | 10.4 | Vegetable and animal oils and fats |
| 11. | 10.5 | Dairy products |
| 12. | 10.6 | Grain mill products, starches and starch products |
| 13. | 10.7 | Bakery and farinaceous products |
| 14. | 10.8 | Other food products |
| 15. | 10.9 | Prepared animal feeds |
| 16. | 11.01-6 and 12 | Alcoholic beverages & Tobacco products |
| 17. | 11.07 | Soft drinks |
| 18. | 13 | Textiles |
| 19. | 14 | Wearing apparel |
| 20. | 15 | Leather and related products |
| 21. | 16 | Wood and of products of wood and cork, except furniture; articles of straw and plaiting materials |
| 22. | 17 | Paper and paper products |
| 23. | 18 | Printing and recording services |
| 24. | 19 | Coke and refined petroleum products |
| 25. | 20A | Industrial gases, inorganics and fertilisers (all inorganic chemicals) - 20.11/13/15 |
| 26. | 20B | Petrochemicals - 20.14/16/17/60 |
| 27. | 20C | Dyestuffs, agro-chemicals - 20.12/20 |
| 28. | 20.3 | Paints, varnishes and similar coatings, printing ink and mastics |
| 29. | 20.4 | Soap and detergents, cleaning and polishing preparations, perfumes and toilet preparations |
| 30. | 20.5 | Other chemical products |
| 31. | 21 | Basic pharmaceutical products and pharmaceutical preparations |
| 32. | 22 | Rubber and plastic products |
| 33. | 23OTHER | Glass, refractory, clay, other porcelain and ceramic, stone and abrasive products - 23.1-4/7-9 |
| 34. | 23.5-6 | Cement, lime, plaster and articles of concrete, cement and plaster |
| 35. | 24.1-3 | Basic iron and steel |
| 36. | 24.4-5 | Other basic metals and casting |
| 37. | 25OTHER | Fabricated metal products, excl. machinery and equipment and weapons & ammunition - 25.1-3/25.5-9 |
| 38. | 25.4 | Weapons and ammunition |
| 39. | 26 | Computer, electronic and optical products |
| 40. | 27 | Electrical equipment |
| 41. | 28 | Machinery and equipment n.e.c. |
| 42. | 29 | Motor vehicles, trailers and semi-trailers |
| 43. | 30.1 | Ships and boats |
| 44. | 30.3 | Air and spacecraft and related machinery |
| 45. | 30OTHER | Other transport equipment - 30.2/4/9 |

Table 2. Commodities and industries (cont.)

| ONS industry code | | Industry description |
|---|---|---|
| 46. | 31 | Furniture |
| 47. | 32 | Other manufactured goods |
| 48. | 33.15 | Repair and maintenance of ships and boats |
| 49. | 33.16 | Repair and maintenance of aircraft and spacecraft |
| 50. | 33OTHER | Rest of repair; Installation - 33.11-14/17/19/20 |
| 51. | 35.1 | Electricity, transmission and distribution |
| 52. | 35.2-3 | Gas; distribution of gaseous fuels through mains; steam and air conditioning supply |
| 53. | 36 | Natural water; water treatment and supply services |
| 54. | 37 | Sewerage services; sewage sludge |
| 55. | 38 | Waste collection, treatment and disposal services; materials recovery services |
| 56. | 39 | Remediation services and other waste management services |
| 57. | 41-43 | Construction |
| 58. | 45 | Wholesale and retail trade and repair services of motor vehicles and motorcycles |
| 59. | 46 | Wholesale trade services, except of motor vehicles and motorcycles |
| 60. | 47 | Retail trade services, except of motor vehicles and motorcycles |
| 61. | 49.1-2 | Rail transport services |
| 62. | 49.3-5 | Land transport services and transport services via pipelines, excluding rail transport |
| 63. | 50 | Water transport services |
| 64. | 51 | Air transport services |
| 65. | 52 | Warehousing and support services for transportation |
| 66. | 53 | Postal and courier services |
| 67. | 55 | Accommodation services |
| 68. | 56 | Food and beverage serving services |
| 69. | 58 | Publishing services |
| 70. | 59-60 | Motion Picture, Video & TV Programme Production, Sound Recording & Music Publishing services & Programming And Broadcasting services |
| 71. | 61 | Telecommunications services |
| 72. | 62 | Computer programming, consultancy and related services |
| 73. | 63 | Information services |
| 74. | 64 | Financial services, except insurance and pension funding |
| 75. | 65 | Insurance and reinsurance, except compulsory social security & Pension funding |
| 76. | 66 | Services auxiliary to financial services and insurance services |
| 77. | 68.1-2 | Real estate services, excluding on a fee or contract basis and imputed rent |
| 78. | 68.2IMP | Owner-Occupiers' Housing Services |
| 79. | 68.3 | Real estate services on a fee or contract basis |
| 80. | 69.1 | Legal services |
| 81. | 69.2 | Accounting, bookkeeping and auditing services; tax consulting services |
| 82. | 70 | Services of head offices; management consulting services |
| 83. | 71 | Architectural and engineering services; technical testing and analysis services |
| 84. | 72 | Scientific research and development services |
| 85. | 73 | Advertising and market research services |
| 86. | 74 | Other professional, scientific and technical services |
| 87. | 75 | Veterinary services |
| 88. | 77 | Rental and leasing services |
| 89. | 78 | Employment services |
| 90. | 79 | Travel agency, tour operator and other reservation services and related services |
| 91. | 80 | Security and investigation services |
| 92. | 81 | Services to buildings and landscape |
| 93. | 82 | Office administrative, office support and other business support services |

**Table 2. Commodities and industries (cont.)**

| ONS industry code | | Industry description |
|---|---|---|
| 94. | 84 | Public administration and defence services; compulsory social security services |
| 95. | 85 | Education services |
| 96. | 86 | Human health services |
| 97. | 87-88 | Residential Care & Social Work services |
| 98. | 90 | Creative, arts and entertainment services |
| 99. | 91 | Libraries, archives, museums and other cultural services |
| 100. | 92 | Gambling and betting services |
| 101. | 93 | Sports services and amusement and recreation services |
| 102. | 94 | Services furnished by membership organisations |
| 103. | 95 | Repair services of computers and personal and household goods |
| 104. | 96 | Other personal services |
| 105. | 97 | Services of households as employers of domestic personnel |
| | | **Government provided services** |
| 106. | 38g | Waste collection, treatment and disposal services; materials recovery services non-market |
| 107. | 49.3-5g | Land transport services and transport services via pipelines, excluding rail transport non-market |
| 108. | 52g | Warehousing and support services for transportation non-market |
| 109. | 59-60g | Motion Picture, Video & TV Programme Production, Sound Recording & Music Publishing services & Programming And Broadcasting services non-market |
| 110. | 84g | Public administration and defence services; compulsory social security services non-market |
| 111. | 85g | Education services non-market |
| 112. | 86g | Human health services non-market |
| 113. | 87-88g | Residential Care & Social Work services non-market |
| 114. | 90g | Creative, arts and entertainment services non-market |
| 115. | 91g | Libraries, archives, museums and other cultural services non-market |
| 116. | 93g | Sports services and amusement and recreation services non-market |
| | | **Non-profit institutions serving households** |
| 117. | 64-69n | Financial (64-66), Real Estate (68.1-68.3) and Legal (69.1) Services NPISH |
| 118. | 72n | Scientific research and development services NPISH |
| 119. | 75n | Veterinary services NPISH |
| 120. | 81n | Services to buildings and landscape NPISH |
| 121. | 85n | Education services NPISH |
| 122. | 86n | Human health services NPISH |
| 123. | 87-88n | Residential Care & Social Work services NPISH |
| 124. | 90n | Creative, arts and entertainment services NPISH |
| 125. | 91n | Libraries, archives, museums and other cultural services NPISH |
| 126. | 93n | Sports services and amusement and recreation services NPISH |
| 127. | 94n | Services furnished by membership organisations NPISH |

*Note:* Commodities appearing in red belong to the tourism bundle; see section 11.

## Table 3. Steps to convert raw data in model compatible form

**Step 1**

- Raw data converted from Excel to header array format.
- Data is labelled with brief commodity/industry labels.

**Step 2**

- Any missing values are replaced by informed guesses consistent with other data.
- Any values with no economic meaning (e.g., negative imports) are set to zero or small positive values.
- Data is reformatted into model form.
- Margin commodities are determined. These are
  - electricity, transmission and distribution;
  - gas; distribution of gaseous fuels through mains; steam and air conditioning supply;
  - wholesale and retail trade and repair services of motor vehicles and motorcycles;
  - wholesale trade services, except of motor vehicles and motorcycles;
  - rail transport services;
  - land transport services and transport services via pipelines, excluding rail transport;
  - water transport services.
- Data is checked for balance.

**Step 3**

- Data is relabelled with brief names.
- Imports are checked so that they do not exceed total domestic consumption.
- A variety of diagnostic checks are performed.

**Step 4**

- Margins are scaled so that they do not total domestic consumption.

**Step 5**

- Margins and taxes are allocated across domestic and imported flows.

**Step 6**

- Gross operating surplus is divided across non-labour factors of production.
- Wages are divided across occupational categories using labour hours data.
- Employment and labour supply data by occupation is added.

See Figure 5 for diagrammatic exposition of the first two steps listed above.

**Step 7**

- Investment inputs by industry are introduced.
- Import-domestic elasticities of substitution are introduced.

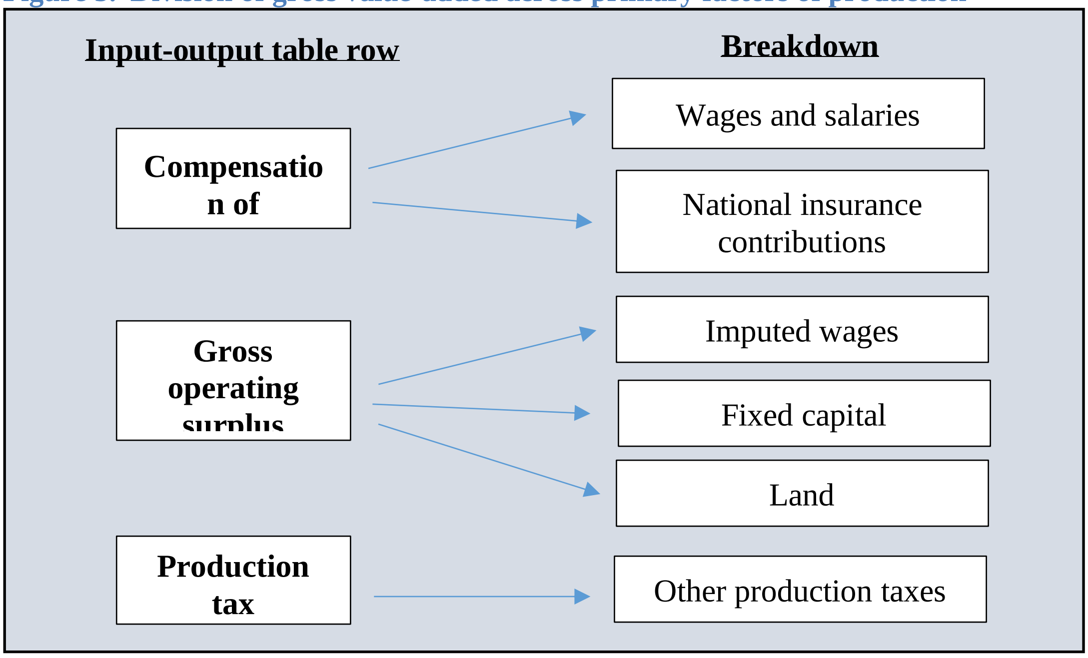
**Step 8**

- Investment inputs are divided across imported and domestic flows.
- First version of model data files created.

**Table 3. Steps to convert raw data in model compatible form (cont.)**

| |
|---|
| **Step 9**<br>• Income taxes are determined.<br>• Investment-capital ratios and rates of return are determined.<br>• Government expenditure on non-consumption items (i.e., investment and transfer payments) are determined.<br>• Stocks are adjusted to equal zero for services.<br>• Balance of payments data are determined.<br>• The final version of all model data and parameter files are created.<br>• Data imbalances caused by the second and fourth steps listed above are removed by allocating them to an other costs category. |
| **Step 10**<br>• Other costs are shocked to zero via a model simulation thus removing data imbalances while maintaining all balancing conditions. |
| **Step 11**<br>• Model data is updated to 2016 national accounts values via a model simulation. |
| **Step 12**<br>• Model data is divided across the four countries. This applies shares to divide<br>o output;<br>o household consumption;<br>o investment;<br>o government consumption;<br>o exports;<br>o employment;<br>o labour supply.<br>• The shares are based on four country data available from the ONS website. |

**Figure 5. Division of gross value-added across primary factors of production**



## 18. Household demand parameters

### 18.1. Theoretical background

Section 10 describes the demand system applied to represent household preferences: LES. This system requires the calibration of parameters that capture the response of consumers to changes in relative prices and to income, i.e., price and income elasticities. These responses will vary depending on the nature of the commodity. Here we describe how these parameter values were estimated.

To estimate behavioural parameters for the LES we use the Twice-Extended Linear Expenditure System (TELES). The TELES is a version of the LES developed by Tulpule and Powell (1978) with the intention of reducing the number of unknown parameters so as to facilitate estimation and thus overcome the problem of multicollinearity. TELES is so-named as it represents the LES extended twice: once for savings and once for leisure.

The TELES is represented by the following (simultaneous) equations

$$P_i X_i = P_i \mu_i + \beta_i \left( \alpha - \sum_{i=1}^{n} P_i \mu_i + w_i \mu_H \right) \qquad (i = 1,\ldots, n), \tag{45}$$

$$S = \beta_s \left( \alpha - \sum_{i=1}^{n} P_i \mu_i + w \mu_H \right), \tag{46}$$

$$-wH = -w\mu_H + \beta_L \left( \alpha - \sum_{i=1}^{n} P_i \mu_i + w \mu_H \right), \tag{47}$$

where

$P_i$ is the price of good *i*,

$X_i$ is the quantity consumed of good *i*,

$S$ is saving in dollars,

$\alpha$ is after-tax non-labour income,

$w$ is the hourly after-tax wage rate,

$H$ is hours worked per person,

$\mu_i$ is subsistence consumption of good *i*,

$\mu_L$ is minimum acceptable leisure hours,

$\mu_H$ is maximum acceptable work hours (total hours less $\mu_L$),

$\beta_i$ is the marginal propensity to consume good *i* out of full income,

$\beta_S$ is the marginal propensity to save out of full income,

$\beta_L$ is the marginal propensity to consume out of full income.

The TELES model defined by (45)-(47) is linear in prices and income but multiplicative in $\mu_i$ and $\beta_i$. Thus the estimation of the model is somewhat more complex than in the simple linear case. As total expenditure and income are equivalent here, the sum of the disturbances for each equation is equal to zero. Thus when estimating the model the covariance matrix will be singular and the estimation process will fail. This problem is overcome by omitting one arbitrary equation that is then deducible via the adding-up restriction (Judge *et al*., 1988). In this case, following Tulpule and Powell (1978) equation (46) is omitted by assuming that the subsistence value of leisure equals zero. This leaves the parameters $\beta_i$, $\beta_L$, $\mu_i$ and $\mu_H$ to be estimated. The marginal propensity to save out of full income $(\beta_S)$ can then be recovered from the normalisation condition: $\sum_{i=1}^{n} \beta_i + \beta_L + \beta_S = 1$.

### 18.2. Data and estimation

We utilise detailed quarterly time series data on national product, income and expenditure from the United Kingdom Economic Accounts (UKEA) published by the Office for National Statistics (2017) to estimate the parameters described above.[12] Quarterly data is likely to reflect less-than-full adjustment of demand to changes in income and prices over such a short time horizon. This may have implications for the estimates generated. Quarterly data on private final consumption expenditure at current prices and price indices for the period Quarter 1, 1988 to Quarter 1, 2017 for 12 broad commodity groups are taken from the UKEA. These estimates are provided in trend and seasonally adjusted terms. Prior to publishing quarterly time series income and expenditure data, the ONS identifies, estimates and removes regular seasonal fluctuations and calendar effects associated with the time of the year from these time series data. Where trend and seasonally adjusted estimates are not available, original data are provided.

The 2017 UKEA provides a snapshot of UK households' spending based on the latest Classification of Individual Consumption According to Purpose (COICOP). The COICOP classifies individual consumption-related data connected to household budget surveys on goods and services by categories of purpose incurred by three institutional sectors: households, general government and non-profit institutions serving households. The COICOP classifies household expenditure into 12 groups (see Table 4). The consumption expenditure data comprises the monetary value of all goods and services purchased by households. Table 4 presents household final consumption expenditure shares from Quarter 1 2015 to Quarter 1 2017.

**Table 4. UK household final consumption expenditure shares**

| Commodity groups | 2015 | | | | 2016 | | | | 2017 |
|---|---|---|---|---|---|---|---|---|---|
| | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | Q3 | Q4 | Q1 |
| 1. Food and non-alcoholic beverages | 0.086 | 0.085 | 0.081 | 0.083 | 0.083 | 0.081 | 0.079 | 0.081 | 0.083 |
| 2. Alcoholic beverages, tobacco & narcotics | 0.039 | 0.038 | 0.038 | 0.040 | 0.038 | 0.038 | 0.037 | 0.040 | 0.037 |
| 3. Clothing and footwear | 0.051 | 0.056 | 0.055 | 0.066 | 0.051 | 0.055 | 0.055 | 0.068 | 0.052 |
| 4. Housing; water; electricity; gas and other fuels | 0.280 | 0.255 | 0.244 | 0.250 | 0.276 | 0.253 | 0.242 | 0.250 | 0.269 |
| 5. Furnishings; household equipment and routine maintenance of the house | 0.045 | 0.047 | 0.047 | 0.052 | 0.047 | 0.048 | 0.047 | 0.052 | 0.048 |
| 6. Health | 0.017 | 0.017 | 0.017 | 0.018 | 0.018 | 0.018 | 0.017 | 0.018 | 0.018 |
| 7. Transport | 0.138 | 0.141 | 0.156 | 0.124 | 0.140 | 0.140 | 0.154 | 0.119 | 0.137 |
| 8. Communication | 0.020 | 0.019 | 0.019 | 0.020 | 0.020 | 0.020 | 0.020 | 0.020 | 0.021 |
| 9. Recreation and culture | 0.091 | 0.098 | 0.098 | 0.104 | 0.096 | 0.101 | 0.100 | 0.105 | 0.098 |
| 10. Education | 0.016 | 0.016 | 0.016 | 0.016 | 0.017 | 0.016 | 0.016 | 0.016 | 0.017 |
| 11. Restaurants and hotels | 0.086 | 0.099 | 0.106 | 0.095 | 0.087 | 0.100 | 0.107 | 0.095 | 0.088 |
| 12. Miscellaneous goods and services | 0.130 | 0.128 | 0.123 | 0.133 | 0.127 | 0.129 | 0.127 | 0.136 | 0.133 |
| Total | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

*Source:* The Office for National Statistics, *United Kingdom Economic Accounts*, edition no. 98.

[12] The original quarterly data are available from the ONS website at https://www.ons.gov.uk/economy/grossdomesticproductgdp/datasets/unitedkingdomeconomicaccounts.

The UKEA also includes estimates of total gross income for households, total disposable income and taxes on income for the period Quarter 1 1988 to Quarter 1 2017. Estimates of the total number of employed persons and average hours worked per week were collected from the latest sources of ONS data on earnings: these were *Average Weekly Earnings*, *Annual Survey of Hours and Earnings*, and the *Labour Force Survey*. The total hours worked are estimated as the product of average hours per employed person, total number of employed persons and number of weeks per quarter. Finally, quarterly estimates of the hourly wage rate are calculated as the ratio of after tax income to total hours worked.

An efficient way to estimate a full system of demand equations is to use a three stage least squares (3SLS) procedure, or a full information maximum likelihood (FIML) estimator. The estimator uses information about all the equations in the system thus providing consistent estimates. The FIML method is used here to solve the demand equations (45) and (47) simultaneously to estimate expenditure elasticities.

The solution of the demand system using FIML gives numerical estimates of the marginal budget shares $(\beta_i)$ that can be employed to estimate the expenditure elasticities for the 12 aggregated commodities taken from the UKEA. Using the estimates of $\beta_i$, the expenditure elasticities $(\eta_i)$ are determined as $\eta_i = -\beta_i \omega$ where $\omega$ is the income elasticity of the marginal utility of income (the 'Frisch' parameter). Note that $\eta_i > 0 \;\; \forall i$ rules out the possibility of inferior commodities. This is a property of the Stone-Geary utility function as it is additive in log form; thus, the marginal utility of good *i* is dependent only on consumption of good *i* and is unrelated to the consumption of any other good. This assumption is appropriate when estimating demand for broad categories of goods (Clements *et al.*, 2020).

The Frisch parameter is defined as the income elasticity of the marginal utility of income. Conceptually, the parameter also represents the ratio of total expenditure to luxury expenditure (Clements *et al.*, 2020). Groom and Maddison (2019) conduct a meta-analysis of five separate empirical methods of estimating the Frisch parameter for the UK and find that a value of -1.5 is defensible for the UK. We adopt this value to determine the income elasticities $\eta_i$. Note that $\eta_i$ represents the initial income elasticities in all four countries. But, as income grows over the simulation horizon the income elasticities will diverge slightly across countries due to divergent income growth. The value of $\omega$ will also diverge from its initial value over the simulation horizon due to income growth.

## 18.3. Elasticity estimates

Applying the FIML method estimates are generated of the TELES parameters. These are combined with $\omega$ to estimate the income (or expenditure) elasticities for the 12 commodity groups. The estimated average budget shares, marginal budget shares, and expenditure elasticities are presented in Table 5.

**Table 5. Estimated TELES parameters and expenditure elasticities**

| Commodity groups | Average budget share | Marginal budget share | Expenditure elasticity |
|---|---|---|---|
| 1. Food and non-alcoholic beverages | 0.082 | 0.036 | 0.438 |
| 2. Alcoholic beverages, tobacco & narcotics | 0.039 | 0.043 | 1.100 |
| 3. Clothing and footwear | 0.057 | 0.030 | 0.521 |
| 4. Housing; water; electricity; gas and other fuels | 0.256 | 0.182 | 0.709 |
| 5. Furnishings; household equipment and routine maintenance of the house | 0.048 | 0.034 | 0.708 |
| 6. Health | 0.017 | 0.023 | 1.330 |
| 7. Transport | 0.139 | 0.060 | 0.434 |
| 8. Communication | 0.020 | 0.017 | 0.861 |

| | | | | |
|---|---|---|---|---|
| 9. | Recreation and culture | 0.099 | 0.058 | 0.590 |
| 10. | Education | 0.016 | 0.039 | 2.388 |
| 11. | Restaurants and hotels | 0.097 | 0.332 | 3.429 |
| 12. | Miscellaneous goods and services | 0.129 | 0.146 | 1.130 |

The results show that food, transport, and clothing and footwear have the lowest expenditure elasticities. Furthermore, as the expenditure elasticities for food, transport, clothing and footwear, recreation and culture, furnishings, housing etc., and communication are all less than one implies these commodities are necessities for UK households. For the remaining commodities, the expenditure elasticities are greater than one implying that these commodities are luxuries.

## 19. Import-domestic elasticities of substitution

### 19.1. Theoretical background

In the model described above, domestically-produced and imported goods are assumed to be imperfectly substitutable due to their heterogeneous nature. This assumption applies for all users of these commodities: households, firms and the government. This treatment of domestically-produced and imported goods has been adopted in the literature to incorporate the cross-hauling observed in international trade data. That is, the phenomenon that countries simultaneously import and export the same commodities. The degree of heterogeneity will vary by commodity and country and is reflected by the elasticity of substitution: higher elasticity values imply less heterogeneity, lower values imply greater heterogeneity. Here we describe the elasticity values estimated.

Each commodity available to users is a constant-elasticity-of-substitution (CES) aggregate of the domestically-produced and imported varieties. The utility-maximising problem of the representative consumer for a given level of total expenditure for good *i* $\left(Y_i\right)$ is

$$Max\ U_i = \left[\alpha_i D_i^{\frac{\sigma_i-1}{\sigma_i}} + \left(1-\alpha_i\right) M_i^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i-1}} \tag{48}$$

$$\text{subject to } Y_i = p_i^D D_i + p_i^M M_i \qquad \forall i, \tag{49}$$

where

$U_i$ is the utility of the representative consumer,

$D_i$ and $M_i$ is demand for domestic and imported goods,

$\sigma_i$ is the elasticity of substitution between domestic and imported good *i*,

$\alpha_i$ is a distribution parameter, and

$p_i^D$ and $p_i^M$ are the prices of domestic and imported good *i*.

Using the objective function and budget constraint above we can derive the first-order conditions for the least-cost combination of domestic and imported good *i*

$$\frac{M_i}{D_i} = \left(\frac{1-\alpha_i}{\alpha_i}\right)^{\sigma_i} \left(\frac{P_i^D}{P_i^M}\right)^{\sigma_i}. \tag{50}$$

Equations (50) are the demand functions for the problem specified by the objective function and budget constraint; they say that a one per cent increase in the price ratio of domestic and imported good *i* will cause a $\sigma_i$ per cent increase in the ratio of imported and domestic good *i*. These demand functions are applied to estimate $\sigma_i$ .

## 19.2. Specification of the model

Here we describe the statistical model and its constraints that lead to the most direct and popular way of estimating the elasticity of substitution between domestic and imported goods. We apply a log-linear functional form because it results in estimated elasticities that are constant and therefore allows direct estimation of the desired elasticities. The following two log-linear equations represent the demand for domestic and imported goods:

$$\ln D_i = \alpha_{0i} + \alpha_{1i} \ln \overline{Y} + \alpha_{2i} \ln P_i^D + \alpha_{3i} P_i^M + \alpha_{4i} \ln \overline{P}, \tag{51}$$

$$\ln M_i = \beta_{0i} + \beta_{1i} \ln \overline{Y} + \beta_{2i} \ln P_i^D + \beta_{3i} P_i^M + \beta_{4i} \ln \overline{P}, \tag{52}$$

where $\overline{Y}$ and $\overline{P}$ are nominal GNP and aggregate price of all goods. The parameters to be estimated are $\alpha_{0i}$ - $\alpha_{4i}$ and $\beta_{0i}$ - $\beta_{4i}$.

Subtracting (52) from (51) gives the demand functions in relative form

$$\ln\left(\frac{D_i}{M_i}\right) = \theta_{0i} + \theta_{1i} \ln \overline{Y} + \theta_{2i} \ln P_i^D + \theta_{3i} P_i^M + \theta_{4i} \ln \overline{P}, \tag{53}$$

where $\theta_i \left(= a_i - \beta_i\right)$ is the parameter to be estimated. The disadvantage of this relative functional form is that the parameters $\alpha_i$ and $\beta_i$ cannot be identified from $\theta_i$. But the advantage is that the relationship between $D_i$ and $M_i$ is more stable than individual demand equations (Leamer and Stern, 1970).

Imposing homogeneity, i.e., $\sum_{i=1}^{4} \alpha_i = \sum_{i=1}^{4} \beta_i = \sum_{i=1}^{4} \theta_i = 0$, on (53) gives

$$\ln\left(\frac{D_i}{M_i}\right) = \theta_{0i} + \theta_{1i} \ln\left(\frac{\overline{Y}}{\overline{P}}\right) + \theta_{2i} \ln\left(\frac{P_i^D}{\overline{P}}\right) + \theta_{3i} \ln\left(\frac{P_i^M}{\overline{P}}\right). \tag{54}$$

Applying the assumption of a constant elasticity of substitution for each good *i*, i.e., $\alpha_{2i} = -\alpha_{3i}$, $\beta_{2i} = -\beta_{3i}$, $\theta_{2i} = -\theta_{3i}$, to (54) gives

$$\ln\left(\frac{D_i}{M_i}\right) = \theta_{0i} + \theta_{1i} \ln \overline{Y} + \theta_{3i} \ln\left(\frac{P_i^M}{P_i^D}\right) + \theta_{4i} \ln \overline{P}. \tag{55}$$

Applying the restriction that the income expansion path for domestic and imported good *i* is identical, i.e., $\theta_{1i} = 0$ or $\alpha_{1i} = \beta_{1i}$, gives

$$\ln\left(\frac{D_i}{M_i}\right) = \theta_{0i} + \theta_{3i} \ln\left(\frac{P_i^M}{P_i^D}\right). \tag{56}$$

Equation (56) is the most common form used to estimate the elasticity of substitution between domestic and imported goods. In (56) $\theta_{3i}$ is the elasticity of substitution and is expected to be positive.

The values for the dependent variable $\ln\left(\frac{D_i}{M_i}\right)$ were constructed as

$$\ln\left(\frac{D_i}{M_i}\right) = \ln\left(\frac{Q_i}{P_i^Q} - \frac{X_i}{P_i^X}\right) - \ln\left(\frac{M_i}{P_i^M}\right), \tag{57}$$

where

$Q_i$ and $P_i^Q$ are the value and price of domestic production of good *i*,

$X_i$ and $P_i^X$ are the value and price of exports of good *i*,

$M_i$ and $P_i^M$ are the value and price of imports of good *i*.

### 19.3. Data and results

To estimate the substitution elasticities data were collected on the prices of imports, exports and domestic goods, and the volume of imports, exports and domestic sales of domestically-produced goods from the Office for National Statistics (https://www.ons.gov.uk). The available data on import and export price indices, and import and export volume indices have a consistent commodity classification. But the data on domestic sales of domestically-produced goods and domestic price indices have slightly different classifications; the majority of the sectors were similar and were matched accordingly.

To keep the estimation model simple and aid interpretation of results, the original data were aggregated to a more manageable number of sectors. The data series were aggregated to 17 sectors, the majority of which are manufacturing industries: see Table 6.

**Table 6. Estimation results**

| Industry | Elasticity of substitution | | R-Squared | Durbin-Watson statistic |
|---|---|---|---|---|
| Products of agriculture | 1.46 | *** | 0.93 | 1.94 |
| Forestry and fishing | 1.70 | *** | 0.71 | 1.96 |
| Mining and quarrying | 1.59 | *** | 0.96 | 2.01 |
| Preserved meat, meat products and fish | 1.36 | *** | 0.99 | 2.05 |
| Dairy products | 1.80 | *** | 0.93 | 2.07 |
| Other food products | 1.17 | *** | 0.88 | 1.89 |
| Alcoholic beverages | 1.95 | *** | 0.92 | 2.16 |
| Soft drinks, mineral waters | 0.86 | *** | 0.70 | 2.13 |
| Tobacco products | 1.21 | *** | 0.87 | 1.83 |
| Textiles, wearing apparel and leather products | 2.47 | *** | 0.98 | 1.85 |
| Wood, and products of wood, cork and printing | 0.99 | *** | 0.98 | 1.99 |
| Coke and refined petroleum products | 1.65 | *** | 0.74 | 1.91 |
| Chemicals and pharmaceutical products | 3.85 | *** | 0.94 | 2.11 |
| Furniture and other manufactured goods | 2.09 | *** | 0.94 | 1.99 |
| Electricity, gas, steam and air conditioning | 1.37 | *** | 0.71 | 1.91 |
| Water supply, sewerage and waste management | 0.74 | | 0.60 | 1.90 |
| Other manufacturers | 0.90 | * | 0.69 | 1.90 |

Note: *** = $p<0.01$, ** = $p<0.05$, * = $p < 0.1$.

Equation (57) was estimated using quarterly data from 1998 to 2016. The Seemingly Unrelated Regression Equations (SURE) model was used to solve a 17-good demand system for imports and domestic substitutes simultaneously. Table 6 reports the estimated elasticities of substitution as well as the diagnostic results.

As expected, all elasticities are positive. Further, all estimates are significant except for *water supply, sewerage and waste management*. Most estimates are between 1 and 2. Further, most values are well away from unity (the Cobb-Douglas value). The estimates range from 0.74 for *water supply, sewerage and waste management* to 3.85 for *chemicals and pharmaceutical products*.

Table 6 also presents a brief summary of the fit of the estimated model. The diagnostic results reveal that the model fit the data well (high R-squared) and there are no major statistical problems. The test for the presence of autocorrelation in the errors of the regression (Durbin-Watson statistic) ranges from 1.83 to 2.16, indicating no serious autocorrelation.

## References


Atalay, E. (2017), 'How important are sectoral shocks', *American Economic Journal: Macroeconomics*, vol. 9, no. 4, pp. 254–80.

Bargain, O., Orsini, K. and Peichl, A. (2011), *Labour Supply Elasticities in Europe and the US*, Discussion Paper 5820, IZA, Bonn, June.

Bruno, M. (1984), 'Raw materials, profits, and the productivity slowdown', *The Quarterly Journal of Economics*, vol. 99, no. 1, pp. 1–30.

Chirinko, R.S. (2008), 'σ: The long and short of it', *Journal of Macroeconomics*, vol. 30, issue 2, pp. 671–86.

Clements, K.W., Mariano, M. and Verikios, G. (2020), 'Estimating the linear expenditure system using cross-sectional data', Department of Economics Working Paper, The University of Western Australia (forthcoming).

Clements, K.W., Selvanathan, S. and Selvanathan, E.A. (1995), 'The economic theory of the consumer', in Selvanathan, E.A. and Clements, K.W. (eds), *Recent Developments in Applied Demand Analysis: Alcohol, Advertising and Global Consumption*, Spinger Verlag, Berlin, pp. 1–72.

Dixon, P., and Rimmer, M. (2002), *Dynamic General Equilibrium Modelling for Forecasting and Policy. A Practical Guide and Documentation of MONASH*, Elsevier, Amsterdam.

Geary, R.C. (1950), 'A note on 'a constant-utility index of the cost of living'', *Review of Economic Studies*, vol. 18, no. 2, pp. 65–66.

Groom, B. and Maddison, G. (2019), 'New estimates of the elasticity of marginal utility for the UK', *Environmental and Resource Economics*, vol. 72, issue 4, pp. 1155-82.

Hanoch, G. (1971), 'CRESH production functions', *Econometrica*, vol. 39, no. 5, pp. 695–712.

Hertel, T.W. and Tsigas, M.E. (1997), 'Structure of GTAP', in Hertel, T.W. and Tsigas, M.E. (eds), *Global Trade Analysis: Modeling and Applications*, Cambridge University Press, Cambridge, pp. 13–73.

Horridge, J.M. (2011), *The TERM Model and Its Data Base'*, General Paper G-219, Centre of Policy Studies, Monash University, July.

Harrison, W.J. and Pearson, K.R. (1996), 'Computing solutions for large general equilibrium models using GEMPACK', *Computational Economics,* vol. 9, no. 2, pp. 83–127.

HM Revenue & Customs (2013), *HMRC's CGE Model Documentation*, HM Revenue & Customs, London, December.

Judge, G., Hill, R.C., Griffiths, W.E., Lutkepohl, H. and Lee, T.C. (1988), *Introduction to the Theory and Practice of Econometrics*, 2nd edition, John Wiley & Sons, New York.

Klein, L.R. and Rubin, H. (1948), 'A constant-utility index of the cost of living', *Review of Economic Studies*, vol. 15, no. 2, pp. 84–87.

Leamer, E.E. and Stern, R.M. (1970), *Quantitative International Economics*, Allyn and Bacon, Boston.

Office for National Statistics (2016), *Input-Output Analytical Tables*, *2013*, London.

Office for National Statistics (2017), *United Kingdom Economic Accounts*, edition no. 98, London.

Starr, R.M. (1997), *General Equilibrium Theory*, *An Introduction*, Cambridge University Press, Cambridge, UK.

Stone, R. (1954), 'Linear expenditure systems and demand analysis: an application to the pattern of British demand', *The Economic Journal*, vol. 64, no. 255, pp. 511–527.

Theil, H. (1980), *The System-Wide Approach to Microeconomics*, The University of Chicago Press, Chicago.

Tulpule, A. and Powell, A.A. (1978), *Estimates of Household Demand Elasticities for the ORANI Model*, Preliminary Working Paper OP-22, Industries Assistance Commission, Canberra, Australia.

Vincent, D.P., Dixon, P.B. and Powell, A.A. (1980), 'The estimation of supply response in Australian agriculture: The CRESH/CRETH production system', *International Economic Review*, vol. 21, no. 1, pp. 221–42.