THE TIME DIVERSIFICATION PUZZLE: A SURVEY

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ABSTRACT

Since Samuelson's (1969) theoretical proof that risk and time are unrelated, a half century of debate and controversy has ensued, leaving time diversification as one of the most enduring puzzles of modern finance. The most conspicuous aspect of the debate is the questionable assumptions that underlie much of the analysis. Thus we are left with an unsatisfying debate conducted in a paradigm where terminal wealth is usually a function only of returns, and where time-weighted measures are assumed to adequately evaluate performance. This paper reviews the major streams in the time diversification literature and argues that more realistic analysis using defensible assumptions is likely to lead to better prescriptions for improved retirement investing.

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1.0 Introduction

The time diversification puzzle is concerned with the relationship between investment risk and investment horizon. Beyond this there appears little else upon which the protagonists agree. In the literature we see a multitude of theoretical priors, schools of thought, quantitative methods, and conceptions of risk and yet we have no resolution to the debate. Why is it that a subject that has attracted some of the most fertile minds in economics remains so elusive? In this paper, we argue that at least part of the problem is that the debate is conducted on the wrong terms. In particular, we argue that there is too narrow a focus on returns as the sole determinant of terminal wealth. Realistic accumulation models are also a function of contributions and salary growth so without correcting the basis upon which the debate is conducted, we argue that it will remain abstract and of limited practical value.

When thinking about investment risk, we anchor our work in Markowitz (1952), who considered the problem of portfolio choice under uncertainty[1]. As with much ground-breaking research, Markowitz's (1952) modern portfolio theory was a caricature of a more complex problem. In particular, Markowitz (1952) considered portfolio choice in a single period setting, allowing him to remove time-variation from the problem of portfolio selection.

Naturally, the financial economics literature evolved and scholars began to consider the portfolio selection problem in a multi-period setting like that encountered with practical investment problems. Chief among these scholars was another Nobel Prize winner – Paul A. Samuelson – who considered the problem in a multi-period setting using expected utility theory. Samuelson's (1969) work is of particular interest for two reasons. Firstly, he was amongst the first to bring the genius of Markowitz's (1952) work into a multi-period setting which by itself is remarkable[2]. Secondly, and particularly germane to this paper, Samuelson (1969) initiates the time diversification debate by considering whether the concept of diversification works with time, in the same way as it does amongst assets or securities (cf. Markowitz, 1952). In order to study the existence of time diversification, Samuelson (1969) selects the classical expected utility theory as his framework of choice. Expected utility theory is thus the point of departure for the time diversification debate, and all other competing streams or schools of thought tend to emerge at least in part as a reaction to Samuelson's (1969) work.

A literature review on time diversification without a discussion about expected utility theory, or any of the other major competing schools of thought, would (and should) be regarded as lacking. In order to synthesise the time diversification debate, we must conduct a critical survey of the literature. Fortunately, without too much effort, a process of taxonomy results in a number of distinct streams or schools of thought. Each of these competing schools tends to coalesce around an alternative theory to, or a common critique of, expected utility theory. The battle of ideas between these schools of thought has led to time diversification becoming one of finance's most enduring puzzles.

1 Markowitz's contribution earned him a share in the 1990 Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel.

2 Others include Tobin (1965) and Merton (1969).

Following the literature more or less chronologically, we commence the formal review of the literature with the expected utility theory stream beginning with Samuelson (1969). In this seminal work, Samuelson (1969) isolates the relationship between risk and time by observing the optimal allocation to risk assets with horizon, based on three assumptions. While a number of proponents confirm the mathematical certainty of his findings, even more scholars – including some who are otherwise advocates of expected utility theory – call into question Samuelson's (1969) assumptions. In fact, it is Samuelson's (1969) three assumptions –which will be scrutinised throughout this paper – that provide later scholars the oxygen to keep the time diversification debate burning.

From the initial stream of work led by Samuelson (1969), the rise of the Black-Scholes-Merton Option Pricing Theory – another Nobel Prize winning idea – offered a convenient basis upon which Bodie (1991, 1995) could observe risk[3]. Bodie (1995) concluded that, because the price of a put option increased with investment horizon, so did risk. The option pricing theory approach of Bodie (1991, 1995) apparently emerged because of this unrelated breakthrough in economics, not as a result of a specific critique of Samuelson's (1969) work. Only later, did others highlight that Bodie's (1995) approach appeared to offer an objective measure of risk, in contrast to Samuelson's (1969) normative treatment[4]. After Bodie's (1991, 1995) early contribution, the option pricing theory school of thought tended to degenerate into semantic debates about whether Bodie (1995) was correctly identifying the price of insurance, and relating it properly to the investment horizon. After a burst of scholarship in the late 1990s, this stream in the time diversification literature has to some degree faded away. Perhaps the most substantial critique of Bodie's (1995) work was that it was conducted in a risk-only framework. In a sense, this is understandable given the relationship of interest in the time diversification debate is the one between risk and investment horizon. On the other hand, opponents questioned whether it is appropriate to separate risk from return, thereby overlooking one of investment's key trade-offs.

It is a generalisation that behavioural economists inevitably end up becoming amongst the most vocal opponents of any framework that tends to see economics as (hard) science, as opposed to social science. These two visions of economics mix like oil and water. Behavioural economists introduce the richness of humanity to economic problems, often in qualitative terms, whereas "scientists", of whom Samuelson (1969) was most definitely one, prefer to take approaches characterised by theoretical formality and the rigour of mathematical reasoning, even if it means making simplistic assumptions about human behaviour. In these few sentences, we have briefly outlined both the behaviouralists' principal critique of Samuelson (1969) – the inappropriateness of his underlying assumptions – and our critique of the behavioural stream of literature – the lack of framework, and negative approach to the problem. From the authors' observation of the literature, the behaviouralists tend to avoid the formal frameworks of economics in analysing the

[3] Robert C. Merton and Myron S. Scholes shared the 1997 Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel. Fischer Black died of cancer in 1995. Because the Prize in Economic Sciences is not awarded posthumously, Black is not formally recognised as a laureate although his contribution is beyond question.

[4] Recall, that to study risk by estimating the optimal allocation to risk assets, Samuelson (1969) first had to assume the investor's form of risk aversion.

question at hand. By way of analogy, behaviouralists could be described as the insurgents of the time diversification debate. They appear on the scene, they challenge the entrenched position with the insights of psychology, only to retreat without leaving a tangible alternative framework. From the literature review conducted in this paper, we will see that the influence of the behaviouralists is thus limited to providing critiques of the other streams of the literature. Notwithstanding its limitations, the behavioural stream in the literature does provide some compelling insights, particularly relating to the selection of risk measures for examination.

The final stream in the literature – what Booth (2004) describes as the “applied” stream – is defined more by what it’s not, than what it is. While the applied stream is a somewhat nebulous confection of studies, there is the faint semblance of a unifying theme. Scholars who pursue this path tend to approach the problem of time diversification empirically, and without resting on a theoretical edifice as the more established streams tend to. Simulation techniques are also a common methodological choice as Booth (2004) suggests. Parallel to the time diversification debate, has emerged a rich literature on risk measures. Leaning on this literature, applied scholars tend to define risk in a certain way – for example, value-at-risk – and then proceed to estimate their selected risk measure over a number of horizons of different lengths. Scholars then draw conclusions about the presence or otherwise of time diversification by applying reasoning to these estimates. Naturally, it is possible to define risk in many ways and so the applied stream has tended to grow as new conceptions of risk emerge. As will become evident from this paper, some scholars have even developed measures purely for the purposes of analysing the time diversification question.

So, almost fifty years after the debate began, we continue to see studies emerge which seek to resolve the debate. As noted, several recent studies have even introduced particular measures whose sole purpose is to shed light on this debate. And yet the puzzle remains unresolved. It would thus be fair to say that there are a number of entrenched camps who are unable to agree on a common basis from which to the advance the argument. This paper does not seek to induce the debate’s antagonists to recant. Rather, by leveraging off aspects of each stream in the literature, and attempting to synthesise the debate, we set out to offer an alternative view of the relationship between risk and investment horizon. Thus, as has happened to the option pricing theory stream to some extent, sides in the debate are lost to time. This Kuhnian (1970) process reminds us of a famous quotation which, ironically enough, is attributed to Samuelson: “Funeral by funeral, theory advances (Wilson, 1998, p.52).”

This paper is about the relationship between investment risk and investment horizon, not about the objective merits of expected utility theory, or any of the other schools of thought we discuss. We accept there are other economic problems where any one of these frameworks might represent the superior framework within which to approach the given problem. Our interest here is whether the given framework – for example, expected utility theory – is the best approach to gleaning positive insights into the puzzle of time diversification. We posit that our critique should therefore be viewed narrowly through the lens of the purpose of this paper.

We agree with Kritzman (2000) that, as far as the debate thus far goes, everyone is right on their own terms. The entrenched camps do not seem to be budging from their position. But given the magnitude of financial assets associated with long-horizon investing, and the continued shift to defined contribution style plans, we argue that scholars do not have the luxury of conducting semantic debates about abstract ideas. We argue that the first step is to locate the debate in reality but incorporating realistic accumulation models in analysis, and then to apply the flexible empirical approaches preferred by the applied stream in the literature. By doing this we believe that practitioners can apply insights from scholarly research to improve outcomes for their plan members.

Before we begin our detailed examination of the literature, we will first summarise the evolution of ideas that led to the time diversification debate as necessary context. The literature review thus commences with a discussion of the work of Markowitz (1952).

2.0 The time diversification puzzle

2.1 Markowitzian origins

The principle concern of the time diversification debate is the relationship between investment risk and the investor's horizon. But before we explore the subject in detail, we must first anchor the work in the field of finance. Fortunately, when it comes to investment – or, in formal terms, portfolio choice – we can rely on one of the most famous theories in all of finance: the pioneering work of Markowitz (1952).

Markowitz's (1952) seminal modern portfolio theory (MPT) was the first formal treatment of the benefits of portfolio diversification. The theory showed that, by constructing a portfolio of imperfectly correlated assets, it was possible to reduce portfolio risk for a given level of expected return. Formally, a portfolio, *p*, is said to be mean-variance efficient, or a superior portfolio, if it produces greater return for a given level of risk than any other portfolio, *q*, of the same assets, that is,

$$E(R_p) > E(R_q) \text{ where } \sigma_p^2 = \sigma_q^2 \qquad [1]$$

or less risk for a given level of return,

$$\sigma_p^2 < \sigma_q^2 \text{ where } E(R_p) = E(R_q) \qquad [2]$$

assuming the same assets. The two parameters for portfolio, *p*, of *n* assets are derived as follows:

$$E(R_p) = w'R \qquad [3]$$

$$\sigma_p^2 = w' \Sigma w \qquad [4]$$

where w' and w are, respectively, the row and column vectors of the portfolio weights of *n* assets, *R* is the vector of expected returns for *n* assets, and Σ is the matrix of variances and covariances between the *n* assets. The parameters for portfolio, *q*, are defined analogously.

While the MPT of Markowitz (1952) remains one of the iconic theories of finance, it is founded on a number of assumptions which have been critiqued extensively in the literature. Of these assumptions, the most relevant to this literature review is its single-period character. MPT's single-period assumption implies that the optimisation procedure need only be performed once because it is based on single, static estimates of the optimisation's inputs – the variables in equations [3] and [4]. The single-period assumption implicitly relies on two important points: firstly, that the investor sees the life of the portfolio as one period; and, secondly, that the investor has in mind estimates of return and risk that characterise the performance of each asset for the entire period in question, along with a fixed covariance (or correlation) matrix that captures the co-movement between assets for the period.

In reality, investors view portfolio decisions as iterative. Realised returns and revised expectations necessitate a review of portfolio objectives, or the estimates used in constructing the portfolio (e.g. Grinold and Kahn, 1999). For example, further portfolio decisions may be required for the following reasons: the portfolio has failed to achieve the investment objective(s) set for it; it has achieved its objective(s) with too much risk; the investment objectives have changed; or, perhaps most importantly, expectations about the future performance of the underlying financial assets have changed. These kinds of examples give us a realistic idea of the context in which portfolio choice decisions takes place.

It is thus apparent that MPT's single-period assumption is seriously challenged by the realities of investing, with today's institutional setting being vastly more complex than that assumed by the basic MPT model, or by the time diversification debate. The critique of the single-period assumption is that it is too simplistic, and incompatible with the context in which real portfolio decisions are made. Multi-period approaches to the portfolio problem also have a relatively long history in the finance literature, and it is in a multi-period setting that this debate is squarely located (Tobin, 1965). Moreover, not only does the time diversification debate consider a multi-period problem, it is essentially about the relationship between multiple investment periods of different lengths – the "investment horizon" – and risk.

Beginning with Samuelson (1969), financial theorists considered whether it was possible to reduce portfolio risk by investing over successively longer periods[5]. Is it possible to reduce portfolio risk by spreading risk across *t* time periods, rather than *n* assets? Over time, the answering of this

5 Some might be tempted to date the birth of the time diversification debate to Samuelson (1963). In this paper, Samuelson (1963) clarifies a mistaken interpretation of Bernoulli's Law of Large Numbers where it is thought that "an insurance company reduces its risk by increasing the number of ships it insures (p. 50)." In correcting this interpretation, Samuelson (1963) states that risk is not reduced by adding new risks (as in additional gambles, time periods, or ships insured), it is reduced by subdividing risks (as insurance companies and portfolio managers do). In clarifying one mistaken interpretation he appears to unwittingly introduce a dichotomy where we are choosing between adding a risk or subdividing a risk. While the problem Samuelson (1963) addressed was no doubt of interest, it has on occasion distracted scholars from the real dichotomy at the heart of the time diversification debate. For example, Oldenkamp and Vorst (1997), in an option pricing framework, compare the performance of a ten-year strategy to a one-year strategy repeated ten times. In Samuelson's (1963) terms, this is an example of subdividing risk. Oldenkamp and Vorst (1997) confidently state: "Thus, there are many scenarios in which the repeated one-year strategy outperforms the long-horizon strategy (p. 58)." We believe that Merrill and Thorley (1997) dispelled this misguided notion once and for all when, in critiquing Oldenkamp and Vorst (1997) directly, they note: "The objective in the time diversification debate is to compare risk at *different* (original emphasis) time horizons, not the same horizon (p. 62)." In the interests of complete clarity, we concur with Merrill and Thorley (1997) and see time diversification as a debate about the relative risk of horizons of different length. The comparative performance of strategies with the same horizon, but different reinvestment frequencies, would appear to be more relevant to something like the term structure literature.

question has become the time diversification puzzle, a debate that has been nourished by a focus on long-horizon investing born of the growth of private retirement savings.

Time diversification was first examined in an expected utility framework by Samuelson (1969) who found that the allocation to risky assets is independent of time, and only determined by risk tolerance. These conclusions were based on three assumptions:

1) the investor exhibits constant relative risk aversion;

2) returns follow a random walk[6]; and

3) wealth is a function only of returns.

Much of the subsequent research within the expected utility framework has considered variations to these assumptions, and the competing streams of research use these assumptions as a critique of the framework itself. Without overstating our case, these assumptions are absolutely central to any debate relating to time diversification.

Of Samuelson's three assumptions, the first and second have provided the motivation for much of the subsequent literature. As foreshadowed above, the time diversification literature can be neatly classified into four streams or schools of thought: (1) the expected utility theory stream; (2) the option pricing theory stream; (3) a behavioural finance stream; and, (4) an applied stream. The balance of this paper will outline, and discuss in detail, these separate streams in the time diversification literature which tend to dwell on Samuelson's (1969) first and second assumptions. But before we proceed to the literature review proper we will briefly discuss how Samuelson's (1969) three assumptions represent the battle ground of this debate.

2.2 Samuelson's first and second assumptions

As this paper will show, much of the time diversification debate has revolved around Samuelson's (1969) first two assumptions – that the investor exhibits constant relative risk aversion, and returns follow a random walk – with the debate taking place both within, and between, the four streams in the literature.

The classical approach requires the researcher to outline how they propose to isolate the relationship between risk and time, motivated by the literature. Predictably, the approach taken corresponds to the researcher's preferred paradigm, or their theoretical priors. For example, a proponent of the expected utility theory would typically estimate the allocation to risky assets that maximises expected utility, given a set of assumptions, and relate their results to investment horizon. Another approach might adopt a different framework, a different set of assumptions, and/or a different methodology. Studies therefore vary on a number of dimensions meaning it is difficult to isolate the essential relationship between risk and investment horizon because each difference in dimension introduces its own variation. We can therefore say that researchers have expended much more effort on adding to the debate – by considering a slightly different definition of risk, or a different set-up – than on synthesising or distilling the debate in search of generality. This

[6] The idea that returns on financial assets are random or don't follow a predictable pattern.

willingness to re-visit the debate on different terms, and potentially arrive at a different conclusion, led Kritzman (2000) to make his memorable remark about the time diversification debate becoming a futile referendum on risk. Thus, what is needed is a study that distils the existing literature in search of durable truths. Armed with these truths – if, in fact, such truths exist – we are better positioned to bring them to bear on the essential economic activity motivating this paper: portfolio choice.

2.3 Samuelson's final assumption

Samuelson's (1969) final assumption – that wealth is a function only of returns – is the subject of remarkably little investigation in the time diversification literature. Most scholars are comfortable making Samuelson's (1969) final assumption theirs too. Why this is the case can only be conjectured, but it is thought to relate to the fact that an initial endowment model allows the researcher to ignore factors which both complicate their analysis, and are difficult to generalise in establishing a hypothetical investor. Unfortunately, in accepting this assumption scholars effectively divorce their research from the institutional setting in which long-horizon portfolio choice takes place. In today's world, we observe long-term savings generated by returns, contributions and the associated compounding. We are thus left with studies that are devoid of context; indeed, from a parallel universe.

Where scholars do pursue alternatives to the initial endowment accumulation model, they tend to consider their results in isolation. Thus, we see the results these models produce, but without the benchmark of the initial endowment model, we are none the wiser about how varying the accumulation model affects the relationship between risk and investment horizon. That the time diversification debate remains unresolved, and the marginal effect of alternative contributions is under-studied, is remarkable given that at this very point in time there are trillions of dollars of retirement savings being invested[7]. Without answers to these questions, we are left to wonder upon what basis these investment decisions are being made.

As we know, Samuelson's (1969) final assumption holds that wealth is only a function of returns. We also know that returns are, in turn, a function of the portfolio of assets held. Throughout much of the literature, asset allocation is a constant with stocks generally dominating portfolio composition. Given the central role of stocks to investing, this is not surprising. Furthermore, if we are to observe the essential relationship between risk and time, we would seek to limit additional sources of variation in the results and might therefore hold portfolio composition constant. However, since the time diversification debate began, approaches to asset allocation have evolved appreciably. The balanced fund design has been replaced by, or supplemented with, target date funds and a newer generation of dynamic fund designs. These newer fund designs – given they all have non-constant asset allocations – implicitly seek to expose the investor to risk at the appropriate time. These designs also implicitly acknowledge that there is an important difference between return- and wealth-based conceptions of performance. So, according to modern portfolio design principles, the timing, as well as the magnitude of returns, influences investment

7 Pensions & Investments (2012) estimates the total assets of the world's largest one thousand retirement plans at $US6.7 trillion, as at 30 September 2011.

performance. Once again, we see that the terms upon which the debate is conducted are divorced from the prevailing institutional setting.

3.0 The time diversification literature

As we have noted, Samuelson's (1969) seminal work on time diversification was founded on three assumptions: (1) the investor exhibits constant relative risk aversion, (2) returns follow a random walk, and (3) wealth is a function only of returns. Of these three assumptions, the first and second have motivated almost all the subsequent time diversification literature. We consider these two assumptions together in one section because many previous studies in the literature discuss and vary both assumptions simultaneously.

In order to make this review easier to navigate we divide the literature into a number of streams which have emerged based on common critiques, or consistent theoretical or methodological foundations. Each stream is introduced in chronological order by the date of publication of the stream's foundational work. The exception to this rule is the most diverse stream in the literature – the applied stream – which is introduced last because it leans on elements of each of the other streams, and adopts diverse approaches to studying the time diversification phenomenon. Our discussion of Samuelson's (1969) final assumption follows separately.

3.1 Samuelson's first and second assumptions

3.1.1 Expected utility theory

The expected utility theory stream of the literature tends to observe risk indirectly. Rather than defining risk a particular way (e.g. standard deviation), then measuring it over various investment horizons, scholars within the expected utility framework typically solve for the optimal allocation to stocks for a given assumption set. In this way, it is possible to impute what happens to risk as horizon changes by observing the optimal allocation to risky assets for a variety of horizons. But in order to analyse risk in this way, Samuelson (1969), and his successors in this stream of literature, assign a risk aversion specification to their hypothetical investor via the selection of a utility function. The direction of the logic thus begins by defining the investor's risk tolerance and then works toward what this means for the relationship between risk and investment horizon. Such an approach therefore tends to be normative because it generalises how investors perceive risk first, and then observes the relationship of interest. This paper will show that the normativity of the expected utility framework, as well as its sensitivity to the specification of risk aversion, represent its key weaknesses.

We contend that the population of investors is highly heterogeneous, and not as amenable to generalisation as advocates of the expected utility framework would have us believe. The very existence of the field of behavioural finance is testament to the fact that much of classical economics assumes away the human aspects of what remains a social science. This critique is not new, and has been used in the literature to motivate both the option pricing framework and behavioural streams in the literature.

Samuelson (1969), in his multi-period generalisation of Markowitz (1952), found, assuming the investor exhibits constant relative risk aversion (CRRA), that "... the optimal portfolio decision is independent of wealth at each stage and independent of all consumption-saving decisions leading to a constant [risky asset weight] *w** (p. 244)." A page later, Merton (1969), in continuous time, confirmed Samuelson's constant weight finding in the presence of CRRA, and extended his analysis to consider a constant absolute risk aversion (CARA) assumption[8]. Merton (1969) found that, assuming CARA, the dollar value of wealth invested in the risky asset remains constant, so as wealth rises the proportion falls. While Merton (1969) admits this form of utility function is behaviourally less plausible, we see that very soon after the time diversification debate began we have evidence that Samuelson's (1969) findings are sensitive to his framework's assumptions.

Kritzman and Rich (1998) clearly show in their Exhibit 2 – reproduced herein as Figure 1 – that the allocation to risky assets is sensitive to how each of Samuelson's (1969) first two assumptions are specified. For example, for each of three asset return processes Kritzman and Rich (1998) consider, we can see that it is possible that the allocation to risky assets may be constant, increase with time, or decrease with time depending on the utility function specification. While we can't completely discard the expected utility framework as a means of analysing the time diversification debate, we are beginning to question how it is possible to rely on conclusions so sensitive to their underlying assumptions.

Figure 1: The impact of preferences and return characteristics on time diversification

This figure reproduces Exhibit 2 (p. 68) from Kritzman and Rich (1998) that shows how the allocation to risky assets varies with utility function, risk aversion and the asset return process.

Utility Specification	Absolute Risk Aversion	Relative Risk Aversion	Impact of Time on Equity Allocation		
			Random Walk	Mean Reversion	Mean Aversion
Log Utility = In(Wealth)	Decreasing	Constant	Hold Constant	Hold Constant	Hold Constant
Square Root Utility = (Wealth)$^{1/2}$	Decreasing	Constant	Hold Constant	Increase	Decrease
Power Utility = -1/Wealth	Decreasing	Constant	Hold Constant	Increase	Decrease
Quadratic Utility = 25 x Wealth + 0.1 x Wealth^2	Increasing	Increasing	Decrease	Decrease	Decrease
Combination Utility = 1/Wealth + In(Wealth)	Decreasing	Decreasing	Increase	Decrease	Increase

8 Merton's (1969) paper literally begins on the next page of the same edition of the Review of Economics and Statistics. Merton (1969) notes that Samuelson's (1969) work is more general with respect to the probability distributions it can handle.

Even the most prolific scholars within this stream of the literature – like Samuelson (1963, 1969, 1971, 1989, 1990, 1994) and Kritzman (1994, 2000) – concede that their general findings against time diversification may not hold with alternative utility specifications[9]. Samuelson (1989), for example, states that if the logarithm of wealth less some subsistence level of consumption is the expected value to be maximised, then a lower allocation to stocks with age can be justified. Samuelson (1989) states:

> "Suppose, though, human nature is such that we are each most anxious not to fall below a 'subsistence' level of terminal wealth [original emphasis] - so that log(W - S) and not log W is the utility whose Expected Value we seek to maximize. In that case [the] contention is correct that older people will put less into risky stocks when they have fewer years to go before the terminal date of retiring or bequeathing (p. 11)."

Similarly, Kritzman (1994) conditions his findings in favour of Samuelson's (1969) original conclusion by highlighting five "valid reasons why you might still condition your risk posture on your investment horizon (p. 17)." One of these reasons is the potential for an investor to have a discontinuous utility function[10]. Kritzman (1994) explains:

> "Consider, for example, a situation in which you require a minimum level of wealth to maintain a certain standard of living. Your lifestyle might change drastically if you penetrate this threshold, but further reductions in wealth are less meaningful. You might be more likely to penetrate the threshold given a risky investment over a short horizon than you would be if you invested in the same risky asset over the long run (p. 17)."

Milevsky (1999) tests just such a discontinuous utility function motivated by the fact that it "has been extolled as conforming to observed investor behavior (p. 271)." Milevsky (1999) supports Samuelson's (1969) results, finding that the optimal allocation to risky assets is independent of time. He also finds that, notwithstanding a constant allocation to risky assets, risk – defined as the probability of earning a cumulative rate of return less than that of the risk-free asset – declines exponentially with investment horizon. Milevsky (1999), thus, differentiates between the risky asset allocation and risk in a way not generally seen in this stream of the literature.

Apart from the aforementioned studies, the expected utility stream of the literature contains numerous other studies which attempt to study time diversification by estimating the optimal allocation to risky assets that maximises expected utility, given some set of assumptions (e.g. Bodie *et al.*, 1992; Levy and Spector, 1996; Levy, 1996; Jagannathan and Kocherlakota, 1996; Van Eaton and Conover, 1998; Hansson and Persson, 2000; Strong and Taylor, 2001; Gollier, 2002; Karlsson,

9 Note that both authors also published other joint work in the time diversification literature, for example, Merton and Samuelson (1974) and Kritzman and Rich (1998), respectively.

10 Kritzman and Rich (1998) reiterate this point regarding a discontinuous utility function and give three practical situations consistent with a sudden drop in utility if a threshold is penetrated: "A decline in the value of pension assets will cause a net pension liability to appear on a company's balance sheet; A covenant on a loan agreement will be breached if assets fall below a specified value; Your spouse will abandon you if your net worth falls by a certain amount (p. 70)."

2006)[11]. What we have shown here is that the central players in this stream in the literature admit that the time diversification debate hinges on the form of risk aversion exhibited by the investor. If the specification changes, as Kritzman and Rich (1998) show us, then so does the relationship between risk and time.

Other streams in the time diversification literature generally motivate their resort to an alternative framework with a critique of the Samuelson (1969) approach. The option pricing protagonists contrast their framework with Samuelson (1969) by highlighting its objectivity and independence from particular models of risk aversion or utility. Fisher and Statman (1999), as representatives of the behavioural stream of the literature, critique the assumptions underlying expected utility theory noting that Samuelson (1969) implies that investors accurately assess the probability of loss, a fact not supported by the behavioural literature. Critiques of expected utility theory even pre-date the time diversification debate itself. For example, in criticising results from a similar framework to that of Samuelson (1969), Roy (1952) comments that "in calling in a utility function to our aid, an appearance of generality is achieved at a cost of a loss of practical significance and applicability in our results. A man who seeks advice about his actions will not be grateful for the suggestion that he maximize expected utility (p. 433)."

As shown in the earlier discussion of Kritzman and Rich (1998), it is common in the expected utility theory literature for authors to vary both of Samuelson's (1969) first and second assumptions simultaneously. This is the principal reason why they are considered together in this paper. For example, Kritzman and Rich (1998) in a matrix, reproduced herein as Figure 1, outline what happens to the optimal allocation to stocks for fifteen separate combinations of utility function – log utility, square root utility, power utility, quadratic utility, and combination utility – and asset return process – random walk, mean reversion, and mean aversion. Once again we see that the verdict on time diversification, from within its foundational paradigm, is highly sensitive to the model specification. Take, for example, the power utility function assumption from Figure 1. Depending on one's view of the asset return process, there are three possible relationships between time and risk.

Another relevant study from the expected utility theory stream of the literature is that of Strangeland and Turtle (1999). It represents another example – like Samuelson (1989) and Kritzman (1994) – where the proponents of expected utility realise that the time diversification puzzle may require alternative, or complementary, approaches to be resolved in any general way[12]. Strangeland and Turtle (1999) state that the presence of time diversification "depends critically on a number of important and highly context-dependent factors (p. 1)." They go on to cite relative risk aversion and the risky asset return process as two, out of a total of six, factors that may affect portfolio choice[13].

[11] Other studies examine optimal allocations from a purely mean-variance perspective including Levy and Gunthorpe (1993), Gunthorpe and Levy (1994), Booth (2004), Mukherji (2008) and Panyagometh (2011). Recent research by Ayres and Nalebuff (2013) compares the performance of a range of investment strategies, including leveraged ones, in what they describe as the "Merton-Samuelson" tradition.

[12] Resolving the time diversification debate for certain specific circumstances is not beyond any of the major approaches taken in the literature. A lack of generality is why time diversification remains a puzzle.

[13] The complete set of factors Strangeland and Turtle (1999) identify are: (1) relative risk aversion; (2) risky asset return process; (3) ability to change work habits; (4) frequency of required withdrawal for the investor's portfolio; (5) existence of non-tradable assets (e.g. human capital); and, (6) changes in investment knowledge over the investor's life.

This context-dependency leads Strangeland and Turtle (1999) to conclude that "... there is little motivation to debate the general merit of time diversification for a *typical* (original emphasis) investor (unless we have a very clear understanding of a typical investor) (p. 1)." We can see that, like us, Strangeland and Turtle (1999) detect a fundamental incompatibility between the restrictive and deterministic nature of their framework on the one hand, and the heterogeneity of the hypothetical investor, and the inconclusive evidence on the process driving asset returns, on the other. But perhaps Strangeland and Turtle's (1999) most revealing comment puts their views beyond doubt. They state that "... the issue of time diversification *cannot* (original emphasis) be completely resolved by resorting to an expected utility framework (p. 2)."

Strong and Taylor (2001), attempting to correct for the restrictive assumptions of earlier studies, examine time diversification with what they describe as "realistic utility functions (p. 268)." Moreover, they claim to "not impose … restrictions on the process followed by risky asset returns (p. 268)." We can see in Strong and Taylor's (2001) intent an implicit recognition of the critiques we make herein: that some of the utility function specifications adopted in the literature are more convenient than representative; and, the verdict on time diversification is, in part, a function of the assumption made regarding the process driving asset returns. At face value, therefore Strong and Taylor (2001) agree with the standard critiques, motivate their work with these critiques, and then set out to provide a corrective for the associated inadequacies. At first it appears that Strong and Taylor's (2001) approach may be sufficiently different to allow a resolution within the expected utility framework. For example, in contrast to Samuelson (1969), Strong and Taylor (2001) "provide support for the practitioner view that equity is a long-term investment," and that there "is evidence that equity represents a (significantly) more desirable investment over a ten-year investment horizon than over a one-month investment horizon (p. 297)." Notwithstanding this finding, we see more evidence of the weaknesses of expected utility theory. Once again, optimal allocations are not robust to "various levels of risk tolerance or various utility functions (p. 298)." Therefore, even "realistic utility functions (p. 268)" don't assist us in unravelling the time diversification puzzle. We, once again, find ourselves searching for an objective measure of risk which allows us to avoid the deterministic models of risk aversion integral to the expected utility stream of the literature.

One commonality between how the expected utility literature deals with the risk aversion specification and the asset return process is that both are approached from a deterministic perspective. Studies typically choose a risk aversion and asset return process assumptions motivated by the relevant literature, and then proceed to analyse that chosen set-up. Unfortunately however, as with utility functions, the views of scholars on the asset return processes driving financial data are mixed. Early research, for example, concluded that stock prices contain a predictable component over short horizons, contrary to Samuelson's (1969) random walk assumption (Bodie, 1976; Jaffe and Mandelker, 1976; Nelson, 1976; Fama and Schwert, 1977). Later studies reported evidence of negative serial correlation, or mean reversion, over longer horizons (Fama and French, 1988; Poterba and Summers, 1988; Lo and Mackinlay, 1988). While attempts have been made to explain mean reversion (e.g. Malliaropulos and Priestley, 1999; Poterba and Summers, 1988; DeBondt and Thaler, 1987, 1989), no decisive argument has yet emerged. And to complicate matters further, a number of scholars find evidence against mean reversion (e.g. Richardson and Stock, 1989; Kim *et al.*, 1991; McQueen, 1992; Miller *et al.*, 1994). Thus, we once again see that the time diversification debate is less about the question at hand – the relationship between risk and time horizon – and more about a second-order question relating to the assumptions behind the expected utility framework, in this latter case about what asset return process drives returns.

Whilst the expected utility stream of literature is perhaps the most voluminous and long-lived in the time diversification debate, there are three principal reasons why we believe it does not offer a solution to the puzzle. Firstly, there is no consensus regarding what utility function specification best represents the "average" investor, if such a generalisation were possible. The evidence from the literature in this respect is not convincing. Secondly, as Strong and Taylor (2001) suggest, estimates of the optimal allocation to risky assets are not robust to various levels of risk tolerance or various utility functions. Thus, any generality we seek to obtain can be rejected by providing a counter-example using an alternative risk aversion or utility specification (Rabin, 1952; Booth, 2004). Finally, we contend that expected utility theory is normative in that it imposes a utility function, or model of risk aversion, then proceeds to analyse the relationship between risk and time[14]. While this paper shows a lack of sympathy for the expected utility framework, the fragmented and contradictory findings of the literature to date certainly gives support to our contention that the time diversification debate is in need of synthesis.

3.1.2 Option pricing theory

Bodie (1991) was the first study to depart from the expected utility framework. In his paper, Bodie (1991) goes beyond "the Samuelson-Merton analysis" – which finds that investment horizon should not affect the optimal asset mix – to investigate "the implications of option pricing theory for investment policy of defined benefit pension plans (p. 57)[15]." Other than a desire to test the time diversification question using a different paradigm, Bodie (1991) provides no motivation for this innovation. It is not until later – Merrill and Thorley (1996) to be precise – that advocates of option pricing theory offer it as an objective assessment of the relationship between risk and investment horizon. As shown in the last section, Samuelson (1969) and his successors, generally proxy risk by estimating the optimal allocations to risky assets over different horizons. So how does Bodie (1991) perceive risk? Using Black-Scholes-Merton Option Pricing Theory, he equates risk with the cost of insuring against shortfall risk. In so doing, Bodie (1991) makes a distinction between the probability of shortfall – which he deems a "faulty definition of risk (p. 60)" – and the cost of insurance against shortfall risk which he estimates with option pricing theory. Bodie's (1991) basic conclusion is: "If the objective of pension asset management is to minimise the cost of providing guaranteed benefits, then the longer the time horizon, the *lower* (original emphasis) the proportion of assets that should be invested in stocks (p. 57)." This finding is both at odds with the findings of

[14] Thorley (1995) argues that "critics invoke mathematical models of risk aversion to argue that investors should not succumb to the time diversification 'fallacy.' The premise of [Thorley's] paper is that these arguments are a misapplication of the positive economic paradigm (p. 73)."
[15] Bodie (1991) uses the general descriptor "Samuelson-Merton analysis" because of the multiple contributions each of these scholars has made to this debate. He cites in particular Merton (1971), Merton and Samuelson (1974), and Samuelson (1963, 1989) as examples.

Samuelson (1969) – who suggests that the allocation to risky assets is a function of risk tolerance not investment horizon – and conventional wisdom – which argues that higher allocations to risky assets can be justified at longer investment horizons.

Bodie's (1995) motivation is identical to his earlier work (see Bodie, 1991) in that he sets out to test the "familiar proposition (p. 18)" at the heart of the time diversification debate: that investing in stocks is less risky, the longer the horizon. He argues that, for this proposition to be true, the cost of insuring against underperforming the risk-free rate should fall as the investment horizon lengthens. The principal difference between the two works is that Bodie (1995) tests his findings for two types of asset return process: the random walk assumption as in Samuelson (1969); and, mean reversion. Bodie (1995) confirms the findings of Bodie (1991) that risk, measured using option pricing theory, increases rather than decreases with investment horizon. Thus, having adopted a different theoretical paradigm, Bodie (1991, 1995) produces results that contradict both conventional wisdom and the main findings of the expected utility theory stream of the literature discussed earlier[16]. In this sense, we see that the time diversification debate is developing *between* paradigms, as well as *within* paradigms. This matters to this paper because, if we are to advocate the importance of synthesis in relation to this puzzle, we must first show that a puzzle exists.

A further relevant contribution of Bodie (1995) is a salient reminder that the investment decision which he examines – and which is the subject of this paper – exists within a broader lifetime planning context as discussed in Bodie, Merton and Samuelson (1992). In their paper, Bodie *et al.* (1992) considers whether the presence of labour flexibility affects consumption, saving, and portfolio investment decisions over the lifecycle. According to this broader context, where *total* wealth is the sum of financial capital and human capital, the investment decision is one of several interrelated factors bearing on lifetime financial planning. A worker's lifetime income profile – which, in present value terms, equals human capital – might thus bear on the investment decision.[17] Bodie (1995) – like Samuelson (1989) and Kritzman (1994) – conditions his findings regarding time diversification by referring to this more comprehensive set-up in Bodie *et al.* (1992). Bodie (1995) states that: "Asset allocation for individuals should be viewed in the broader context of deciding on an allocation of *total* (original emphasis) wealth between risk-free and risky assets (p. 20)." Within this broader context, Bodie (1995) finds a potential justification for a downward sloping allocation to risky assets through time (cf. Bodie *et al.*, 1992).

Merrill and Thorley (1996) favour Bodie's (1995) approach because they view it as an "objective way to evaluate the arguments for and against time diversification" that is "independent of any specific model of investor utility or risk aversion (p. 13)." We thus see the first sign of a formal critique of the expected utility framework motivating work within the option pricing stream of the literature. Despite their agreement with Bodie (1995) about option pricing theory's objective evaluation of risk, Merrill and Thorley (1996) use the same option pricing theory to consider two

[16] Bodie (1996) again examines the relationship between risk and investment horizon, this time in a defined benefit framework. Referring to Harlow (1991), he reiterates his critique of using the probability of shortfall as a risk measure noting that "it completely ignores how large the potential shortfall might be (p. 90)."
[17] This point – that lifetime income bears on the investment decision – and reliance on the work of Bodie et al. (1992) is common in the time diversification literature.

types of financially-engineered products – Protected Equity Notes and Self-Funding Market Collars – and find "that longer time horizons reduce the cost of risk elimination, and by implication, risk itself (p. 13)."[18] Once again, we see an example of a study that yields polar-opposite results to another study within the same theoretical paradigm.

While Merrill and Thorley (1996) muddy the findings of the option pricing theory stream of the literature, they make at least three other critical points about the debate. Firstly, in viewing the time diversification literature, Merrill and Thorley (1996) make a distinction between "practitioner-oriented empirical research" and the work of "financial theorists" (p. 13). The work of these financial theorists is in fact the literature that we are examining in this paper. The critique of the financial theoretical literature is that it tends to degenerate into a debate about the theoretical paradigm – e.g. what is the most defensible utility function – which necessarily leads to less focus on the essential relationship between risk and time horizon. Furthermore, many of Merrill and Thorley's (1996) financial theorists, having outlined the "incontrovertible truth (Kritzman, 1994, p. 17)" and the "mathematical truth (Kritzman and Rich, 1998, p. 71)" gleaned from their theories, go on to provide a litany of reasons why their findings – and, this paper would argue, their theory – might prove unreliable. How are we to be convinced by an argument when the theoretical edifice upon which the argument is built is undermined by the theory's principal proponents? At least, one would argue, the "practitioner-oriented empirical research" seeks to free itself as much as possible from the false comfort of theory. It is this "practitioner-oriented empirical" approach – or what Booth (2004) describes as the "applied" stream in the literature – that will be examined last in this paper.

Secondly, Merrill and Thorley (1996) rightly point out that differences of opinion are "often rooted in semantic issues about the meaning of risk (p. 13)." This statement is both a premonition of Kritzman's (2000) comment about the time diversification debate being "a referendum on the meaning of risk (p. 50)", as well as one of the motivations for separate work on this question (see Bianchi, Drew and Walk, 2014b). If we are to have a fair referendum on risk, are we not obliged to conduct it on common terms? Bianchi, Drew and Walk (2014a) argue that it is, and go on to compare a comprehensive array of risk measures from the literature using a consistent methodology.

Third, and finally, Merrill and Thorley (1996) hint at an important insight into the basis upon which performance ought to be evaluated. They indicate that returns-based measures of performance

[18] Dempsey et al. (1996), Zou (1997) and Oldenkamp and Vorst (1997) provide critiques of Merrill and Thorley (1996). Dempsey et al. (1996) highlight what they believe to be a false analogy in Merrill and Thorley's (1996) interpretation of Bodie (1995). Zou (1997) suggests that Merrill and Thorley's (1996) findings may say more about their methodological approach, than it does about time diversification. Oldenkamp and Vorst (1997) claim to "show that Merrill and Thorley's (1996) conclusions are not as obvious as they claim" and "that their arguments do not resolve the time diversification debate (p. 57)." We note that Merrill and Thorley (1997) provide a qualitative response to Oldenkamp and Vorst's (1997) critique of their work. They state, and we agree, that "the objective in the time diversification debate is to compare risk at *different* (original emphasis) time horizons, not the same horizon. The fact remains that it costs an investor less to insure against underperforming the risk-free rate at a long horizon compared to a short horizon (p. 62)." We make this point about the confusion between investment horizon and rebalancing frequency earlier when discussing the birth of the time diversification debate and Samuelson's (1963) earlier work. Please refer to footnote 5.

may be the wrong basis for evaluation when they state: “Some critics of time diversification object to the use of annualised returns in measuring risk and return across different time horizons and suggest that dollar figures are more revealing. Thus, the argument for time diversification is even stronger when stated, perhaps mistakenly, in simple dollar terms (p. 17).” We argue that, far from being mistaken, wealth-relative terms are the only way to evaluate risk and return, particularly when we depart from the initial endowment model so popular in the literature (see Bianchi, Drew and Walk, 2014b).

As discussed earlier, Bodie (1995) found that “the cost of the insurance rises with [investment horizon] *T* (p. 20)” and, therefore, so does risk, suggesting that time diversification, as conventional wisdom conceives it, does not exist. Numerous scholars have lined up to critique Bodie’s (1991, 1995, 1996) findings, many doing so in a qualitative fashion. Ferguson and Leistikow (1996) question Bodie’s (1995) singular focus on risk instead noting that “if appropriate allocation proportions depend on reward in relation to risk, not just risk, then Bodie’s message for individuals is irrelevant (p. 68).” Taylor and Brown (1996) challenge Bodie’s (1995) analysis on three fronts. Firstly, they suggest that constant relative risk aversion might be a valid assumption over short horizons but may not be over long horizons. [As an aside, this critique would apply equally to Samuelson’s (1969) expected utility framework.] Secondly, Taylor and Brown (1996) argue that Bodie (1995) sets-up a “straw man that he knocks down with unrealistic assumptions (p. 69).” For example, they suggest that the worst-case scenario Bodie (1995) uses is extremely unlikely and is thus unrealistic. And finally, they highlight that Bodie’s (1995) argument fails when his assumption of a constant standard deviation in his application of the Black-Scholes-Merton option pricing model is replaced with a non-constant standard deviation. As the literature shows, very few measures of risk are constant with investment horizon.

Cohen, de Fontenay, Gould, Sirera and Bodie (1996) is a collection of four letters to the editor in response to Bodie (1995). Therefore, other than each responding to Bodie’s (1995) work, there is no common theme between the letters. Rather, they represent a heterogeneous selection of critiques. Cohen, for example, points out that “the reasonable cost of insurance declines as the horizon is extended and/or return expectations are increased (p. 72).” This assertion is supported by Dempsey *et al.* (1996). Gould, who is clearly a practitioner, hints at two points that are of particular interest in this paper: firstly, that “dollar savings (p. 73)” are the most important measure in pension finance problems; and, secondly, that achieving a retirement goal – which Gould expresses in dollar terms – might be a relevant way of conceiving a practical pension finance problem.[19]

Dempsey *et al.* (1996) attempt to synthesise the option pricing literature and clarify the debate. On the one hand, they see, on the basis of option pricing theory, an argument that risk rises with time (e.g. Bodie, 1995) and, on the other, “a conventional insurance premium (p. 57)” argument where risk falls. Dempsey *et al.* (1996) find that Bodie’s (1995) put option prices are correct, but that the price of a put option is a valid measure of riskiness “only in the case in which one may assume that the potential returns for holding the stock in relation to risk do not improve with the investment

[19] Bierman (1997) and Booth (2004) also suggest that the achievement of investment goals might be important.

time horizon (p. 60).” Dempsey *et al.* (1996) conclude that the put option prices of Bodie (1995) “cannot be taken as representing a measure of market risk. The simple reason is that the price of a put option is indicative of *two* (original emphasis) features of the market: risk and the market’s reward for risk that an insurance writer on a stock can expect to achieve (p. 61).”

In conclusion, as Merrill and Thorley (1996) argue, a chief advantage of Bodie’s (1991, 1995) option pricing theory approach to investigating time diversification is that it measures risk objectively, in contrast to the more normative and contested expected utility theory stream of literature. Notwithstanding this advantage, the option pricing stream has been subjected to three specific critiques from both within the paradigm, and in competing paradigms. Firstly, Ferguson and Leistikow (1996) and Bierman (1997) argue that Bodie’s (1995) option pricing approach implicitly ignores reward-for-risk calculations in favour of risk-only ones. Bierman (1997) notes that: “We need to consider an interpretation of risk that includes good outcomes as well as the bad outcomes (p. 52).” In one sense, Bodie’s (1995) risk-only approach is defensible given that time diversification has always been about the relationship between risk and investment horizon. Bierman’s (1997) argument, on the other hand, has merit: if it wasn’t for the returns stocks offer, they wouldn’t be the financial asset *du jour*. The second critique might be appropriately described as a technical argument. As Taylor and Brown (1996) argue, when the constant standard deviation assumption of Bodie (1995) is relaxed, his findings collapse. Thus, in essence, the constant standard deviation assumption is to the Black-Scholes-Merton Option Pricing Theory, as the constant relative risk aversion assumption is to Samuelson’s (1969) expected utility theory. Each is a dubious assumption which proves lethal to its parent theory, and to our attempts to find a general relationship between risk and investment horizon. Third, is the claim by Kritzman and Rich (1998) that, “[u]nfortunately, the option angle of time diversification has resurrected a misguided discussion about the meaning of risk (p. 71).” So not only do we see the debate taking place *between* paradigms, as well as *within* paradigms, we see the recurring theme regarding the meaning of risk. In the option pricing paradigm we see further attempts to isolate the relationship between risk and time, once again from within an incomplete paradigm. The likes of Gould are the rare voices which seek to move the debate from return-only conceptions of risk to more comprehensive and realistic ones expressed in terms of wealth.

3.1.3 Behavioural finance

Until now we have reviewed the two most enduring streams in the time diversification literature: the expected utility theory stream and the option pricing theory stream. Both streams are characterised by strong theoretical foundations, rigorous analytical approaches, and numerous studies. With this third stream – the behavioural finance stream – a different sort of literature is presented. Apart from being relatively new, the behavioural finance literature has neither strong theoretical foundations (of the economic kind), nor any particular analytical approach. There are also few studies in the behavioural finance stream of the literature. Instead, the behavioural finance research applies the insights of psychology to financial decisions in order to better define how to study problems and interpret findings. In this sense, it offers no alternative analytical framework to compete with the two approaches highlighted thus far. Rather it tends to focus on identifying and enunciating deficiencies in the earlier literature. For example, behavioural economists have critiqued the time diversification literature for not framing risk properly.

Olsen (1997) is one study that discusses risk and how it should be framed. In particular, Olsen (1997) points out that risk in pension funds management ought to be considered from the perspective of the plan member, the beneficiary, whose "risk might be related to the loss of a large amount of wealth (p. 62)" versus a manager whose "risk might be associated with a portfolio return below that of one's colleagues (p. 62)[20]." We again see here a distinction between wealth-denominated measures of risk – which Olsen (1997) sees as relevant for pension funds – and return-dominated measures of risk which are arguably more relevant to investment managers. This distinction is a persistent theme in this debate, and the research of scholars like Olsen (1997) provides motivation for the consideration of wealth-denominated measures of performance in empirical studies like Bianchi, Drew and Walk (2014a).

Olsen (1997) also describes risk as a "multiattribute phenomenon (p. 65)" where the principal risk attributes appear to be "the potential for a below-target return, the potential for a large loss, the investor's feeling of control, and the level of knowledge about an investment (p. 65)." The essentially human dimension of these latter two attributes highlights both the contribution of behavioural economists, and the difficulties presented by the qualitative nature of behavioural finance research. On this basis, we will overlook these attributes. The first attribute – the potential for a below-target return - confirms the importance of targets to pension finance problems. Basu, Byrne and Drew (2011) take this point up in the design of their dynamic asset allocation strategy, and their reporting of comparative performance. The second attribute – the potential for a large loss – suggests once again that the magnitude of risk is important, which in turn focuses our attention on wealth-denominated measures of risk[21]. For example, it is a truism that two minus 25 per cent returns are equivalent *in percentage terms* no matter when they occur during a plan member's accumulation phase. If however we compared the impact of equivalent negative returns at two different points in the accumulation phase – say, at age 30 and age 50 – the differences could be materially different *in wealth terms*. Because this paper focusses on the field of pension finance, we take seriously the perspective of plan members, and thus take seriously wealth-denominated performance measures.

Lastly, Olsen (1997) presents evidence that the relative importance of these attributes is a function of "idiosyncratic investor and asset characteristics (p. 65)." In one sense, these findings are not surprising. In another way, the idiosyncratic nature of the relative importance of the attributes represents a telling critique of the standard expected utility theory assumption of constant relative risk aversion. Olsen's (1997) finding suggests that any attempt to generalise risk tolerance may be fraught, notwithstanding its convenience. Olsen (1997) thus lends support to the choice of using objective measures of risk over the normative framework in Samuelson (1969).

[20] In practice, both perspectives are – or at least should be – of interest to pension fund trustees. The plan member perspective is the appropriate terms upon which to consider whether the plan is meeting its commitments to plan members. The latter perspective – the investment manager perspective – is the lens through which evaluation of investment managers should take place. Based on the authors' professional experience, the distinction between these two perspectives is not always appreciated by plan trustees, management or academia.
[21] Rabin (2001) also argues that investors are "loss-averse" rather than risk-averse.

Olsen and Khaki (1998), in discussing how risk is treated in the time diversification literature, make three important points. Firstly, Olsen and Khaki (1998) dismiss expected utility theory as the framework that can resolve the time diversification debate. They note that "the dismissal of time diversification on positive grounds cannot be justified by appeal to the traditional discounted-SEU model (p. 58)." Olsen and Khaki (1998) are indirectly confirming our critique of expected utility theory as normative, because it seeks to impose a model of risk aversion on the hypothetical investor. A further specific critique Olsen and Khaki (1998) make of expected utility theory – or what they describe as "the traditional SEU models" – is that "decision makers do not treat probabilities and outcomes in the multiplicative fashion assumed by the traditional SEU models... decision makers use an additive model of risk (p. 60)." We see here another example of the strengths of the behavioural stream in the literature: a willingness to look beneath the surface of the classical economic models and attempt to reconcile their assumptions with the behaviour of investors. Without this correspondence between the behaviour of investors and economic theory, we risk economic theory becoming an elegant, but not altogether informative, caricature of reality.

Secondly, Olsen and Khaki (1998) continue to provide a range of behavioural insights. Olsen and Khaki (1998) go further than Olsen (1997) and contend that the magnitude of loss is not only an important aspect of risk, it is of paramount importance in understanding risk from the investor's perspective. For example, they note "... investors consider risk a positive function of probability and size of loss, with considerably greater weight being given to the size of the loss than to the possibility of loss (p. 60)." Olsen and Khaki (1998) go on to provide a cautionary note about how the importance of loss relates to probability, emphasising that "... the tendency to ignore low-probability negative outcomes could lead to financial catastrophe. Thus, potential outcomes should not be dismissed as a matter of course when a large portion of one's wealth is at risk (p. 61)." Although large negative returns might be rare, when the portfolio size effect of Basu and Drew (2009a) sees a rapid rise in portfolio wealth as an investor approaches retirement, even a small negative return can result in large impacts on terminal wealth. Insights like this encourage consideration of the full distribution of terminal wealth outcomes, as well as a number of downside risk measures. Olsen and Khaki (1998) even go as far as to question whether time diversification is indeed compatible with a behavioural conception of risk. They note that: "questions remains, however, of whether the concept of time diversification is generally consistent with the concept of risk as it has been documented in other studies of investment behaviour (p. 59)."

Olsen and Khaki's (1998) third, and final, important point is their clear recognition that time diversification is a contested idea, and might therefore reasonably be described as a puzzle. Even at the point in time when their article was published they acknowledged that "the lack of closure on this topic stems from the [economics] profession's failure to accept a common definition of risk (p. 58)." As will become apparent, when we review the applied stream in the literature, there are many more definitions of risk proposed in the literature. Some scholars – in particular those advocating expected utility theory – impose a theoretical framework and, based on a number of assumptions, reach a conclusion regarding the presence of time diversification. Others prefer to estimate risk as objectively, and as empirically, as is possible.

Fisher and Statman's (1999) study has three goals. Firstly, it sets out to explore time diversification's assumptions. In doing so, they confirm many of the earlier critiques of Samuelson's (1969) expected utility theory approach. Fisher and Statman (1999) state plainly: "Samuelson's [1969] mathematics are right, but his assumptions are wrong (p. 90)." Fisher and Statman (1999), in the tradition of this stream of the literature, also contrast the received theory with the behavioural realities. For example, they note in relation to Samuelson (1994) that: "An unstated assumption under the mathematical truth, however, is that investors correctly assess the probabilities of losses. They do not (p. 91)." In a similar way, Fisher and Statman's (1999) second goal is to introduce a "wide range of factors that affect investment choices," beyond risk and return. If one was to take into account factors beyond the investment decision, there are many studies that would assist us (e.g. Bodie *et al.* 1992; Vanini and Vignola, 2002; Gollier, 2002; Cocco *et al.*, 2005). The third, and final, goal of Fisher and Statman (1999) is to "explore the prudence of the time diversification prescription (p. 88)." It is almost as if the authors have grown frustrated with the time diversification debate itself, and instead resort to questioning whether time diversification's classical generalisation – that stocks are less risky over longer horizons – is sensible given the multitude of factors that bear on investment choice. Fisher and Statman (1999) conclude on a wistful and resigned note: "The time diversification debate teaches us little about the relationship between risk and investment horizon, but it teaches us much about the many factors that affect financial choices (p. 96)."

Whilst this behavioural stream in the literature provides useful critiques of particular research – in particular, expected utility theory – it fails to provide any comprehensive or coherent alternative framework for addressing the time diversification puzzle. Instead it provides some useful points: (1) risk should be measured from the perspective of the plan member; (2) risk should be measured relative to the investor's current status not on an absolute level (Booth, 2004); and, (3) that both the magnitude and the probability of loss are relevant considerations in understanding risk. So, while we remain cognisant of the valuable insights of the behavioural literature, a behavioural approach has no answers regarding the puzzle. This is in large part because, as outlined above, the behaviouralists tend to focus on critiques of the existing literature rather than the introduction of new approaches that can be replicated and extended. The behaviouralists principal contribution may be summarised thus: they help nudge the debate away from abstract returns-only thinking to a more comprehensive wealth focus.

3.1.4 Applied approaches

Fourth and finally, there exists what Booth (2004) has described as an "applied" stream of literature that dwells less on theoretical paradigms and more on empirical approaches to addressing the time diversification debate. Booth (2004) states: "In contrast to the theoretical literature, an applied literature has developed based on… simulation [techniques] (p. 3)." These studies generally define risk in a certain way and then turn to measuring that risk over various time horizons in order to identify whether time diversification exists or not. Numerous studies have been conducted and, with time, later authors have sought to synthesise the previous literature often before introducing yet another measure that is claimed to settle the debate (see, for example, Kritzman and Rich, 1998). This led Kritzman (2000) to conclude that "…the time diversification debate, for many, has degenerated into a referendum on the meaning of risk, which is futile

(p. 50)." Until now, however, no one study has examined all these measures using consistent data and methodologies. This lack of consistency means that it is difficult to determine whether the conflicting evidence of time diversification is truly conflicting, or whether it results from a different set-up, or a different range of measures[22]. Before turning to Samuelson's (1969) remaining assumption, this paper must first discuss the important studies in this applied stream of literature, which generally revolve around particular measures of risk.

3.1.4.1 Standard deviation and variations

Consistent with Markowitz's (1952) framework, the time diversification literature first defined risk as the standard deviation of annualised returns and found that, as investment horizon lengthens, risk falls (Bernstein, 1976; Garrone and Solnik, 1976; Lloyd and Haney, 1980; Lloyd and Modani, 1983; McEnally, 1985; Lee, 1990). This finding has been periodically confirmed by later studies that revisit, and attempt to synthesise, the time diversification literature (e.g. Kritzman and Rich, 1998; Kochman and Goodwin, 2002; Guo and Darnell, 2005). McEnally (1985) disagreed that this was an appropriate measure of risk instead arguing that "unpleasant surprises in *total* (original emphasis) returns on terminal values - the values to which the annual rates of return would compound - not surprises in the average *annualised* (original emphasis) rates of return themselves (p. 24)." Using this measure, McEnally (1985) and later authors (e.g. Bernstein, 1985; Leibowitz and Krasker, 1988; Lee, 1990; Peavy and Vaughn-Rauscher, 1994; Kochman and Goodwin, 2001; Hickman *et al.*, 2001; Kochman and Goodwin, 2002; Gollier, 2002) found that, when measured this way, risk rises as investment horizon lengthens.

3.1.4.2 Distribution of outcomes

McEnally's (1985) work is important because it steered the debate away from parametric measures of risk and considered two additional classes of measures: firstly, measures that examine the range of outcomes, and secondly, downside measures of risk. In particular, McEnally (1985) looked at the range of annualised and total returns. Later studies followed McEnally (1985) but examined a range of other measures over various investment horizons. Leibowitz and Krasker (1988) considered the 5^{th}, 25^{th}, 50^{th}, 75^{th} and 95^{th} percentiles of returns. Reichenstein and Dorsett (1995) looked at ending real wealth percentiles, in particular the 5^{th} percentile and the median (or 50^{th} percentile). Thorley (1995) looked at the mean, 10^{th} and 90^{th} percentile of portfolio wealth over five different horizons. Hickman *et al.* (2001) consider the median, in addition to the mean and standard deviation of terminal wealth. Mukherji (2008) looks at the median, the minimum, the maximum, and range of terminal wealth where $1 is invested each month over the investment horizon.

3.1.4.3 Downside risk measures

In studying downside risk, McEnally's (1985) estimated semi-standard deviations (below the mean) for both average annualised and total returns and found that each measure behaved similarly to their standard deviation counterparts. Mukherji (2002, 2008) found similar results using semi-standard deviation, although he refers to it as downside deviation. The next downside risk measure introduced into the time diversification literature was shortfall risk, which is defined as the probability

[22] Bianchi, Drew and Walk (2014b) explore this point.

of falling short of some threshold return (Leibowitz and Krasker, 1988). In most of the literature, the threshold is the return from T-bills, which is generally regarded as the risk-free asset. Leibowitz and Krasker (1988), and later scholars (e.g. Leibowitz and Langetieg, 1989; Butler and Domian, 1991; Leibowitz and Kogelman, 1991; Reichenstein and Dorsett, 1995; Cohen *et al.*, 1996), found that shortfall risk reduces with horizon implying that stocks are less risky over longer horizons. Other authors test these findings using different models. Reichenstein and Dorsett (1995), for example, find that estimates of shortfall risk behave in similar ways for both random walk and mean reversion models. Bierman (1997) uses a binomial model to explore the risks of hypothetical gambles, expressed in shortfall risk terms, as time horizon increases. Thorley (1995) extends the shortfall risk literature by considering the conditional risky option mean (that is, the mean return when it underperforms the risk-free option), as well as the probability of underperforming the risk-free value.

Critics of shortfall risk, like Olsen and Khaki (1998), have argued however that what needs to be taken into account is not only the probability of loss but the *magnitude* of the loss. Leaning on other work of other authors (Diamond 1988; Joag and Mowen 1990; Kaplan and Garrich 1981; Lopes 1995), Olsen and Khaki (1998) argue that not only is the magnitude of loss important, it is given greater weight by investors than the probability of loss. This finding is particularly relevant when we discuss accumulation models which affect portfolio size, for example, where wealth is a function of contributions as well as returns. A downside risk measure that does have the ability to capture the magnitude of a loss is value-at-risk (VaR), which is proposed in the time diversification literature by Panyagometh (2011). In examining VaR and relative VaR he finds that “the risk of loss becomes lower with the increase in the length of the investment period (p. 96).”

3.1.4.4 Risk-adjusted measures of performance

As noted earlier, Bierman (1997), in critiquing Bodie’s (1995) option pricing framework for being narrowly focused on risk, emphasised that reward-for-risk calculations are relevant in understanding portfolio choice problems. That expected risk and reward are positively related is, after all, one of the more durable truths of finance. The time diversification literature includes a number of different measures which seek to understand the reward-for-risk trade-off over various investment horizons. Levy (1972) found that as investment horizon lengthened the estimated Sharpe ratio increased, suggesting a better risk-return trade-off and the presence of time diversification. Levy’s (1972) findings were generally confirmed by later authors (e.g. Lloyd and Modani,1983; Levy, 1984), although Hodges, Taylor and Yoder (1997) find evidence of a hump-shaped profile noting that “...the Sharpe ratio for each portfolio first increases and then decreases as the holding period is extended (p. 77).” Levy (1984) uses the Treynor ratio, a reward-for-systematic-risk measure, and finds that the risk-reward trade-off also improves with horizon for three separate groups of stocks (aggressive, defensive and neutral)[23]. Using the Sortino ratio, Sinha and Sun (2005) find that the reward for downside-risk also improves with time horizon. Mukherji (2002, 2008) reaches similar conclusions using the coefficient of downside deviation, which is essentially the reciprocal of the Sortino ratio. While each of these measures allows us to

[23] Systematic risk in this context, and in the Levy (1984) paper, refers to the capital asset pricing model (CAPM) beta coefficient (β).

consider both the return and the risk aspects of the time diversification puzzle, they each have a weakness in common with shortfall risk. Neither these reward-for-risk ratios nor shortfall risk take into account the increase in the potential magnitude of loss that results from multi-period compounding over a long horizon of, say, 40 years. This deficiency is of greater consequence when one considers problems outside the initial endowment paradigm so prevalent in the time diversification literature.

3.1.4.5 Novel measures of time diversification

Until now, this literature review has only discussed measures that are well known in the broader finance literature. A small number of scholars have developed their own measures in order to shed light on the time diversification puzzle. One measure, *T**, introduced by Guo and Darnell (2005), is defined as "the investment horizon such that the total stock return over this holding period will not become negative at [a given] confidence level (p. 69)." Put another way, using the *T** measure we are, say, 95 per cent confident that we will not have a negative total stock return if the investment horizon is lengthened to *T** years or longer. While this measure is mentioned in the context of the time diversification debate, it doesn't do much to resolve the question of whether time diversification exists or not. Rather, it allows us to compare the risk of alternative asset return processes. Guo and Darnell (2005), for example, find that a mean-reverting process has a lower *T** than a random walk process. This would be the expected result because a random walk process would be expected to have paths which diverge from the mean for longer meaning that, for a given level of confidence, the *T** for a random walk process would be higher (or longer).

Another novel measure, and the last to be considered here, is the time diversification index (TDI) of Fabozzi *et al.* (2006). The TDI is a ratio of normalised risk measures for investment horizons of different length. Fabozzi *et al.* (2006) argue that the strengths of TDI as a measure include that it does not require any specific assumption regarding the risk profile of agents, and it can be computed for any model and any risk measure. For example, if we were to assume that risk is measured by standard deviation, the TDI is essentially calculated by dividing the reciprocal of the Sharpe ratio for the longer horizon by that of the shorter horizon. According to Fabozzi *et al.*'s (2006) rule, time diversification exists where the TDI is less than unity. Using a range of risk measures in calculating the TDI, Fabozzi *et al.* (2006) find little evidence of time diversification.

3.2 Samuelson's final assumption

In order to complete this literature review, it is necessary to address the third and final assumption underlying Samuelson's (1969) expected utility theory which was raised at the outset: that wealth is only a function of returns.

In relation to this third and final assumption, it is important to note that almost the entire time diversification literature takes place within an "initial endowment" framework. Typically, one of two approaches is taken. Firstly, many studies do not mention wealth at all and instead confine themselves to the analysis of accumulation models which are only a function of returns (e.g. Lloyd and Haney, 1980; Lloyd and Modani, 1983; Leibowitz and Langetieg, 1989; Lee, 1990). Alternatively, the study is set-up with some focus on wealth that is a function of only returns and an explicit level of initial wealth (e.g. Marshall, 1994; Reichenstein and Dorsett, 1995; Thorley, 1995; Levy, 1996).

Despite a number of papers making passing references to more realistic and complete models (e.g. Bodie *et al.*, 1992; Kritzman and Rich, 1998), only a very small number of relatively recent studies actually incorporate periodic cash inflows, or contributions, in any way (e.g. Jagannathan and Kocherlakota, 1996; Hickman *et al.*, 2001; Mukherji, 2008; Panyagometh, 2011; Pástor and Stambaugh, 2012; Ayres and Nalebuff, 2013). In addition, a handful of studies analyse cash outflows (or withdrawals from wealth) as a way of studying the interplay between consumption and retirement investing (e.g. Samuelson, 1969; Merton, 1969; Merton and Samuelson, 1974). Thus, the time diversification literature is overwhelmingly dominated by studies where returns are the only determinant of terminal wealth.

The prevailing institutional setting can't support such an assumption. In reality, terminal wealth is a function of not only returns, but of contributions (which in turn are partly a function of salary growth) and asset allocation. The issue of the influence of contributions and asset allocation is taken up in Bianchi, Drew and Walk (2014a) and Bianchi, Drew and Walk (2014b), respectively.

4.0 Summary

Each of the competing streams in the time diversification literature has been the subject of specific criticism. The assumptions underlying Samuelson's (1969) expected utility theory approach have been comprehensively challenged by proponents and opponents alike. Of these critiques, two present themselves as being particularly convincing. Firstly, the requirement to assign a risk aversion specification to our hypothetical investor is normative and, as the behavioural literature shows us, the risk preferences of investors are by no means uniform. This normativity tends to result more in debates about risk preferences, than in discussions about the substance of time diversification: the relationship between risk and investment horizon. And, secondly, in addition to there being little consensus about what risk aversion specification best represents the "average" investor, the conclusions from the expected utility theory framework are not robust to alternative specifications. Furthermore, as Kritzman and Rich (1998) make clear, this sensitivity to specification holds for the asset return process as well. Thus, the verdict on time diversification can change dramatically by merely changing the risk aversion and/or asset return process specification, and any semblance of generality in results is lost.

Option pricing theory has been subjected to three specific critiques from both within the paradigm, and in competing paradigms. Firstly, some scholars argue that Bodie's (1995) option pricing approach implicitly ignores reward-for-risk calculations in favour of risk-only ones. Whilst this focus might be defensible given the substance of the time diversification debate, stocks are a popular investment because with the risk comes (expected) return. Risk/ return calculations should therefore at least be considered. Secondly, as Taylor and Brown (1996) argue, when Bodie's (1995) constant standard deviation assumption is relaxed, his findings fail. The option pricing framework is thus similar to the expected utility theory stream in that both appear sensitive to the major variable in their specification. Finally, as Kritzman and Rich (1998) suggest, Bodie's (1995) "option angle (p. 71)" perpetuates the debate over the meaning of risk. Thus, we see this recurring theme regarding the meaning of risk taking place between paradigms, as well as within paradigms.

The strength of the behavioural finance literature is that it provides timely critiques of the broader literature by reminding scholars that flesh-and-blood investors don't necessarily correspond to the hypothetical investor represented in much of the theory. In particular, it highlights the importance of wealth conceptions of risk over their return-only counterparts. Its principal drawback is that it fails to provide any comprehensive or coherent framework for addressing the time diversification puzzle. Instead it offers some important points to consider when we evaluate risk measures; for example, both the magnitude and the probability of loss are relevant considerations in understanding risk.

The applied stream in the literature is characterised by a modest reliance on economic theory, and an empirical approach to methodology. It also attempts to approach the research questions from within the institutional setting, in contrast to the vast body of time diversification literature which appears detached from, or indifferent to, it. The principle deficiency of the applied stream in the literature is that it has grown as new risk measures emerge without there being any resolution in sight. In this respect, Kritzman's (2000) remark about the time diversification debate degenerating into a referendum on risk remains as true as ever.

Whatever the preferred paradigm, it is almost certainly true that the answer doesn't lie in approaches that abstract from the realities of investing. Investors do not have uniform, easily-caricatured risk preferences, and terminal wealth is not a function only of returns. Those looking to unravel the time diversification puzzle would do well to start in the world as it exists, not as we would like it to exist.

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