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Viscous Attenuation of Interfacial Waves over a Porous Seabed

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ABSTRACT

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In this paper, we study the viscous damping of waves on a stably stratified fluid over a porous elastic seabed. Using the linearised Navier-Stokes equations as the governing equations of motion, the newly derived solution describes the attenuation of a progressive surface wave on a two-layer stratification due to damping caused by interfacial shear and from the wave interaction with a porous elastic seabed. The analytical results are discussed in the light of experimental observations and the viscous effects are considered by comparison with the previous solution for a rigid impermeable seabed. It is shown that the overall decay scale for viscous attenuation is determined by the relative wavelength and the non-dimensional depth of the lower fluid layer. Wave attenuation due to the porous bed is at a maximum when the lower overlying fluid layer is approximately 50% thicker than its Stokes boundary layer.

ADDITIONAL INDEX WORDS: *Interfacial wave, Porous bed, two-layer flow.*

INTRODUCTION

Motivated by the observed damping effects of a soft mud seabed, the dissipation of waves in viscous layered flows has been investigated by several researchers. The two-layer fluid model of MACPHERSON (1980) assumed potential flow in the upper layer and described the viscous lower layer in the form of the linearised Navier-Stokes equations using Voigt's model. It was found that depending on the elasticity and viscosity of the fluidised lower layer the wave attenuation was of the same or larger order of magnitude than the dissipation due to bottom friction or percolation.

DALRYMPLE and LIU (1978) confirmed the experimental observations of GADE (1958) using a two-layer viscous fluid model. They proposed a complete model which was solved by a complex secant method for all depths and an analytical approximation for the case where the lower layer was much thicker than the boundary layer at the rigid bottom. A numerical solution to the complete model was obtained by BALAS (2004) and validated against the experiments of GADE (1958) and SAKAKIYAMA and BIJKER (1989). The boundary-layer model was further extended to the case of nonlinear waves by JIANG and ZHAO (1989).

A closed form analytical solution for the case in which the lower layer is comparable in thickness with the Stokes' boundary layer was derived to the second order in wave steepness by NG (2000). This solution supported the observations of DALRYMPLE and LIU (1978) and GADE (1958) by showing that in addition to the expected increase in wave attenuation with lower layer viscosity, damping effects were most pronounced when the lower fluid layer was not too much denser than the overlying water and was approximately 1.5 times as thick as its Stokes boundary layer.

In this paper we extend the previous studies by considering the problem of a stratified water column above a porous elastic seabed. We study the relative importance of geometric parameters and individual decay mechanisms on the wave amplitude attenuation.

BOUNDARY VALUE PROBLEM

Two-Layer Fluid System

We consider the motion of two stably stratified viscous fluids which are assumed Newtonian and homogeneous. The wave crests are assumed to propagate periodically in the positive x-direction while the z-direction is measured positive upwards from the mudline. The total still water depth of the system, d, is given by the sum of the finite undisturbed depths of the upper and lower layers, h_1 and h_2 respectively.

As depicted in Figure 1, we distinguish between three regions of flow; two fluid layers which are bounded above by a free surface (at $z=d+\eta$), separated by a sharp interface (at $z=h_2+\xi$), and bounded below (at $z=0$) by an infinitely deep seabed which is described using BIOT'S theory of poroelasticity (BIOT, 1941).

We use ρ' and υ' to denote the ratios of the upper fluid density and kinematic viscosity to those of the lower fluid; ρ_1/ρ_2 and υ_1/υ_2, respectively. For a surface wave of length L, $k=k_r+ik_i$ is the unknown complex wave number to be determined where $k_r=2\pi/L$ is the real part of the wave number and the imaginary part, k_i denotes wave damping. It is noted that the real part is ultimately implied in the solutions presented herein.

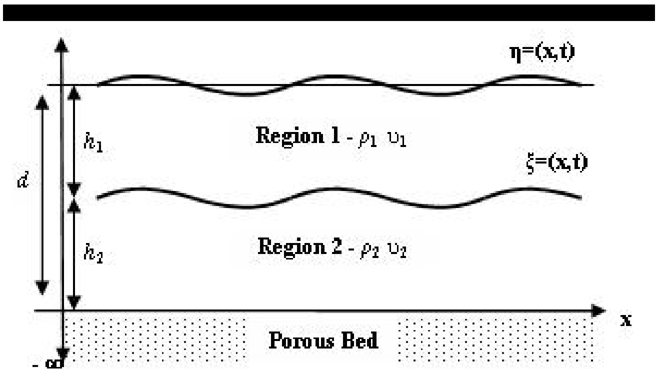

Figure 1. Schematic figure for the two-layer flow model

Governing Equations

Neglecting convective accelerations the incompressible viscous equations of motion for laminar flow in regions 1 and 2 can be expressed as

$$\frac{\partial u_j}{\partial t} = -\frac{1}{\rho_j}\frac{\partial P_j}{\partial x} + \upsilon_j\left(\frac{\partial^2 u_j}{\partial x^2} + \frac{\partial^2 u_j}{\partial z^2}\right) \quad (1)$$

$$\frac{\partial w_j}{\partial t} = -\frac{1}{\rho_j}\frac{\partial P_j}{\partial z} + \upsilon_j\left(\frac{\partial^2 w_j}{\partial x^2} + \frac{\partial^2 w_j}{\partial z^2}\right) \quad (2)$$

And the continuity equation is

$$\frac{\partial u_j}{\partial x} + \frac{\partial w_j}{\partial z} = 0 \quad (3)$$

where P is the dynamic fluid pressure, u and w are the horizontal and vertical velocity components and subscripts j=1,2 indicate the flow region under consideration.

In the porous region we consider the marine sediment as a three phase continuum made up of a solid skeleton of soil particles with interstitial pore space occupied by seawater containing small occluded gas bubbles. Assuming an isotropic bed with compressible pore fluid and a compressible solid matrix, through which the flow in the sediment obeys Darcy's law, the governing equations for fluid motion in the porous region are given by (JENG, 1997);

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} - \frac{\gamma n_e \beta}{K_z}\frac{\partial p}{\partial t} = \frac{\gamma}{K_z}\frac{\partial \varepsilon}{\partial t} \quad (4)$$

$$G\left\{\nabla^2 \zeta + \frac{1}{1-2\mu}\frac{\partial \varepsilon}{\partial x}\right\} = \frac{\partial p}{\partial x} \quad (5)$$

$$G\left\{\nabla^2 \chi + \frac{1}{1-2\mu}\frac{\partial \varepsilon}{\partial z}\right\} = \frac{\partial p}{\partial z} \quad (6)$$

where p is the wave-induced pore pressure, K_z is the vertical coefficient of soil permeability, G is the shear modulus, μ is Poisson's ratio, γ is the unit weight of the pore water, n_e is the soil porosity, ε is the volume strain, defined as

$$\varepsilon = \frac{\partial \zeta}{\partial x} + \frac{\partial \chi}{\partial z} \quad (7)$$

for horizontal and vertical soil displacements of ζ and χ respectively. Coefficient β is the compressibility of the pore-fluid which is defined as

$$\beta = \frac{1}{K'} + \frac{1 - S_r}{P_{wo}} \quad (8)$$

where K' is the true bulk modulus of pore water, S_r is the degree of saturation and $P_{wo.}$ is the average absolute pore water pressure in the sediment.

Boundary Conditions

The linearised boundary conditions to be applied at free surface $z=d$ can be expressed as

$$\frac{\partial \eta}{\partial t} = w_1 \quad (9)$$

$$P_1 - 2\rho_1\upsilon_1\frac{\partial w_1}{\partial z} = 0 \quad (10)$$

$$\rho_1\upsilon_1\left(\frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x}\right) = 0 \quad (11)$$

The continuity of velocities and stresses at elevation $z = h_2$ can be stipulated as

$$\frac{\partial \xi}{\partial t} = w_i \quad (12)$$

$$u_1 = u_2 \quad (13)$$

$$w_1 = w_2 \quad (14)$$

$$P_1 - 2\rho_1\upsilon_1\frac{\partial w_1}{\partial z} - \rho_1 g\xi = P_2 - 2\rho_2\upsilon_2\frac{\partial w_2}{\partial z} - \rho_2 g\xi \quad (15)$$

$$\rho_1\upsilon_1\left(\frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x}\right) = \rho_2\upsilon_2\left(\frac{\partial u_2}{\partial z} + \frac{\partial w_2}{\partial x}\right) \quad (16)$$

Assuming that horizontal flow and bottom boundary layer effects have a negligible effect on the overall seabed response, the continuity of vertical velocity and stresses across the mudline ($z=0$) yields;

$$w_2 = \frac{\partial \chi}{\partial t} - \frac{K_z}{\gamma_w}\frac{\partial p}{\partial z} \quad (17)$$

$$p = P_2 \quad (18)$$

$$\sigma'_z = \tau_{xz} = 0 \quad (19)$$

where τ_{xz} is the wave-induced shear stress, and σ'_z is the vertical effective stress.

Finally, the wave-induced pore pressure and displacements must vanish as the depth of the porous seabed tends to infinity;

$$p = \chi = \zeta = 0 \quad \text{at } z \to -\infty . \quad (20)$$

SOLUTION

Assumed Solution

We assume wave solutions of the form $\eta = ae^{i(kx-\omega t)}$ and $\xi = be^{i(kx-\omega t)}$ where a is the known amplitude of the surface wave and b is the unknown amplitude of the interfacial displacement.

Following the methodology outlined by DALRYMPLE and LIU (1978). When the lower layer is of the same order of magnitude as the boundary layers within the region i.e., $h_2=O(\omega/2\upsilon_2)^{1/2}$, the solutions for the velocities in the fluid layers will be of the form

$$u_j = \frac{i}{k}\frac{\partial w_j}{\partial z} \tag{21}$$

$$w_1 = A\sinh k(z-d) + B\cosh k(z-d) + Ce^{\lambda_1(z-d)} + De^{-\lambda_1(z-h_2)} \tag{22}$$

$$w_2 = E\sinh k(z-h_2) + F\cosh k(z-h_2) + G\sinh\lambda_2(z-h_2) + H\cosh\lambda_2(z-h_2) \tag{23}$$

where parameter λ_j represents the viscosity dominated flow close to the interfacial boundaries and is given by

$$\lambda_j^2 = k^2 - i\omega\upsilon_j^{-1} \tag{24}$$

Matrix formulation

Substituting the above proposed solutions into boundary conditions (9)-(19), and combining the two inhomogeneous free surface boundary conditions [equations (9) and (10)] to obtain one homogeneous boundary condition yields

$$\omega M_1 A - i\rho_1 g(B+C) - 2\omega\rho_1\upsilon_1\lambda_1 C = 0 \tag{25}$$

$$2Bk^2 + (\lambda_1^2 + k^2)C = 0 \tag{26}$$

$$B\cosh kh_1 - A\sinh kh_1 + D = -i\omega b \tag{27}$$

$$Ak\cosh kh_1 - Bk\sinh kh_1 - \lambda_1 D = Ek + \lambda_2 G \tag{28}$$

$$B\cosh kh_1 - A\sinh kh_1 + D = F + H \tag{29}$$

$$M_1(A\cosh kh_1 - B\sinh kh_1) + 2\rho_1\upsilon_1\lambda_1 D = M_2 E - 2\rho_2\upsilon_2\lambda_2 G - (\rho_2 - \rho_1)gb \tag{30}$$

$$\rho_1\upsilon_1\left[2k^2(B\cosh kh_1 - A\sinh kh_1) + (\lambda_1^2 + k^2)D\right] = \rho_2\upsilon_2\left[2k^2F + (\lambda_2^2 + k^2)H\right] \tag{31}$$

$$Ek\cosh kh_2 - Fk\sinh kh_2 + G\lambda_2\cosh\lambda_2 h_2 - H\lambda_2\sinh\lambda_2 h_2 = 0 \tag{32}$$

$$F\cosh kh_2 - E\sinh kh_2 - G\sinh\lambda_2 h_2 + H\sinh\lambda_2 h_2 = \Re\left(E\cosh kh_2 - F\sinh kh_2\right) \tag{33}$$

where

$$M_i = \left(\frac{i\rho_i\omega}{k} - 2\rho_i\upsilon_i k\right) \tag{34}$$

$$\Re = \frac{\rho_2\omega^2}{2Gk^2}(S_a k - (1+2\alpha)S_b + \lambda_p S_c k) - \left\{\frac{i\rho_2\omega K_z}{\gamma_w(1-2\mu)k^2}\left[(1-2\mu-\alpha)kS_b - (1-\mu)(k^2-\lambda_p^2)\lambda_p S_c\right]\right\} \tag{35}$$

and the soil response constants S_i are given by;

$$S_a = \frac{\alpha\{\mu(\lambda_p - k)^2 - \lambda_p(\lambda_p - 2k)\}}{k(k-\lambda_p)(\lambda_p - \lambda_p\mu + k\mu + k\alpha)} \tag{36}$$

$$S_b = \frac{(\lambda_p - \lambda_p\mu + k\mu)}{(\lambda_p - \lambda_p\mu + k\mu + k\alpha)} \tag{37}$$

$$S_c = \frac{k\alpha}{(\lambda_p - k)(\lambda_p - \lambda_p\mu + k\mu + k\alpha)} \tag{38}$$

which are to be determined along with variable k and variable parameters

$$\lambda_p^2 = k^2 - \frac{i\omega\gamma}{K_z}\left\{n_e\beta + \frac{(1-2\mu)}{2G(1-\mu)}\right\} \tag{39}$$

$$\alpha = \frac{(1-2\mu)n_e G\beta}{n_e G\beta + (1-2\mu)} \tag{40}$$

Thus we have nine conditions (25)-(33) for nine unknowns A-H and b. An iterative technique can be used to find a value of k for which the determinant of the matrix of coefficients is zero.

It is noted that when $\Re$=0 the current solution reduces to the rigid impermeable bed solution of DALRYMPLE and LIU (1978).

Solution for Porous Seabed

The general solution for the soil response can be expressed as;

$$\zeta = \frac{ip_o}{2G}\{(S_a + S_b z)e^{kz} + S_c e^{\lambda_p z}\}e^{i(kx-\omega t)} \tag{41}$$

$$\chi = \frac{p_o}{2Gk}\{(S_a + S_b z)ke^{kz} - (1+2\alpha)S_b e^{kz} + \lambda_p S_c k e^{\lambda_p z}\}e^{i(kx-\omega t)} \tag{42}$$

$$p = \frac{p_o}{1-2\mu}\{(1-2\mu-\alpha)S_b e^{kz} + \frac{(\lambda_p^2 - k^2)}{k}(1-\mu)S_c e^{\lambda_p z}\}e^{i(kx-\omega t)} \tag{43}$$

RESULTS AND DISCUSSIONS

In this section, we compare the present solution with the previous analytical solution for a rigid bed beneath a two-layer fluid system (DALRYMPLE and LIU, 1978). We investigate the effects of depth ratio and viscosity on the wave damping and discuss the magnitude of the surface and interfacial wave amplitudes at various lower layer depths.

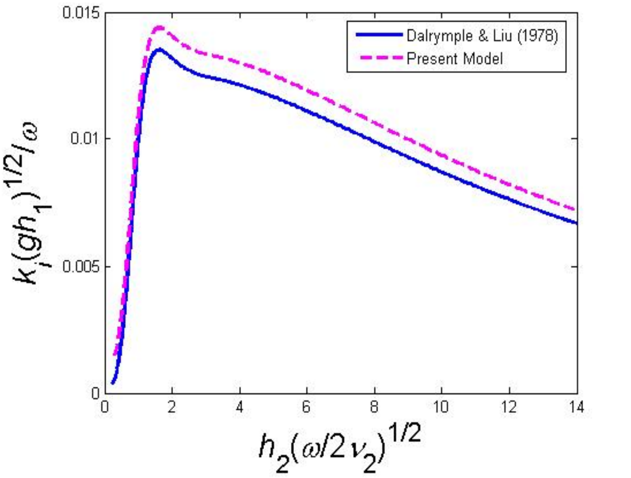

Figure 2. Dimensionless damping coefficient as a function of normalised lower layer depth. (T = 5 s, h_1 = 4 m, ρ_1 = 1028 kg m^{-3}, ρ_2 =1800 kg m^{-3}, h_2 = various, υ_1 = 2.6 x 10^{-6} m^2 s^{-1}, υ_2 = 0.1 m^2 s^{-1}) .

In Figure 2 we compare the dimensionless damping coefficient for the present solution versus that of the rigid bed solution for various values of lower layer depth. The wave and fluid characteristics for this figure were chosen for direct comparison with the results of DALRYMPLE and LIU (1978) and are as follows; T = 5 s, h_1 = 4 m, h_2 = various, ρ_1 =1028 kg m^{-3}, ρ_2 = 1800 kg m^{-3}, υ_1 = 2.6 x 10^{-6} m^2 s^{-1}, υ_2 = 0.1 m^2 s^{-1}. The characteristics of the porous seabed were: n_e = 0.4, S_r= 0.98, G =1 x 10^7 N m^2, μ = 0.33, K_z=10^{-2} m s^{-1}.

From Figure 2 we see that as the lower layer becomes thicker there is an increased efficiency in the ability of the upper layer fluid to overcome the interfacial shear stresses and to do work on the lower layer. This work reaches a maximum and then decreases with increasing fluid depth. As would be intuitively expected the effects of the porous bed on the surface wave damping coefficient are most pronounced as the lower layer of fluid becomes shallow, yet of reasonable thickness with respect to the boundary layer scale. The porous bed solution tends towards that of a rigid bed as the depth of the lower fluid region tend to infinity.

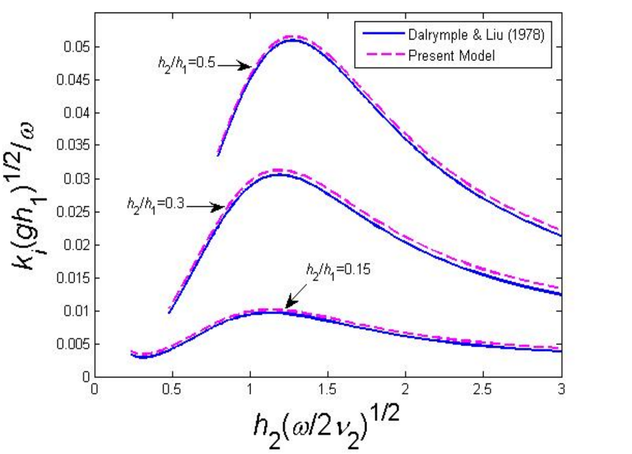

Figure 3. Dimensionless damping coefficient as a function of dimensionless lower layer viscosity for various depth ratios.

(T = 5 s, h_1 = 4 m, h_2 = various, ρ_1 = 1028 kg m^{-3}, ρ_2 = 1800 kg m^{-3}, υ_1 = 2.6 x 10^{-6} m^2 s^{-1}, υ_2 = various)

The experiments of GADE (1958) and SAKAKIYAMA and BIJKER (1989) showed that a peak damping rate occurs when the lower layer of overlying fluid is approximately 30-50% thicker than its Stokes boundary layer. For the rigid bed case given by of DALRYMPLE and LIU (1978), the maximum damping rate occurs where the fluid depth to boundary layer ratio is $(\omega/2\upsilon_2)^{1/2}h_2 \approx 1.5$, which corresponds to a thickness of the lower layer of around 0.6 m. It was found from the present model that the peak damping rate for the porous bed solution occurred at the same lower layer thickness as for the rigid bed, although the magnitude of the attenuation was increased. The significant observation from Figure 6 is that the greatest influence of an underlying porous bed coincides with the maximum damping rate due to shear effects alone, thus accentuating the wave damping in this key region.

In Figure 3 the viscosity of the lower layer is varied for different values of fluid depth ratios h_1/h_2. The greatest difference between the porous and rigid bed solutions occurs as the thickness of the lower layer reduces relative to that of the upper layer, with maximum attenuation occurring when the depth of the lower fluid layer is approximately 1.3-1.5 as thick as its Stokes boundary layer.

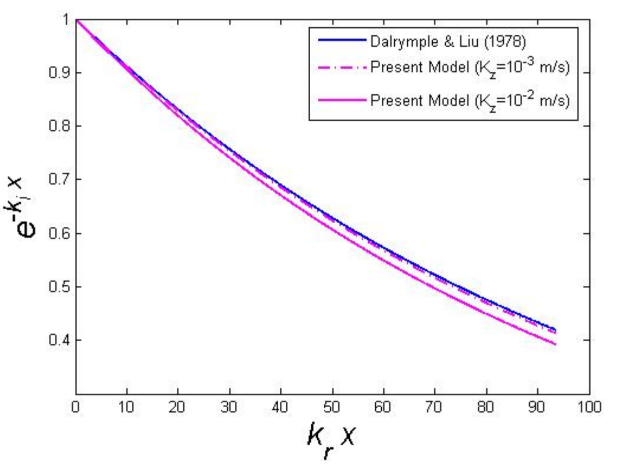

Figure 4. Magnitude of the wave amplitude decay with non-dimensional horizontal propagation distance.

(T = 5 s, h_1 = 4 m, h_2 = 4 m, ρ_1 = 1028 kg m^{-3}, ρ_2 = 1800 kg m^{-3}, υ_1 = 2.6 x 10^{-6} m^2 s^{-1}, υ_2 = 0.1 m^2 s^{-1})

Figure 4 illustrates the wave amplitude decay with non-dimensional propagation distance for porous beds with different vertical permeability coefficient (K_z). It can clearly be observed that for a fine sand ($K_z \approx$ 10^{-3}m/s) the magnitude of the wave amplitude decay is not much greater than that given by the rigid bed solution. However for a coarse sand ($K_z \approx$ 10^{-2}m/s) the presence of the porous bed causes a significant increase in the wave amplitude attenuation.

CONCLUSIONS

A semi-analytical model for a surface wave propagating over a layer of viscous fluid which lies above a porous elastic bed has been presented. This model may be considered as a characterisation of the fluidisation of an oceanic seabed. It was shown that the maximum damping effect of the porous elastic bed coincides with the maximum damping rates due to shear effects, thus accentuating wave damping when the dimensionless lower layer depth is approximately 30-50% greater than the non-dimensional boundary-layer thickness parameter.

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