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Estimating extreme beach erosion frequency from a Monte Carlo simulation of wave climate

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ABSTRACT

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Recent developments in extreme value modelling has been used to develop a robust and defendable framework for determining coastal erosion hazard on sandy coastlines. This framework reproduced well (within factor 2 up to the 15 year return level) the extreme beach erosion volumes obtained from field measurements at Narrabeen Beach, Australia. This encouraging finding was achieved using a simple beach erosion and accretion model following the equilibrium profile approach with exponential response after KRIEBEL and DEAN (1993). The method includes allowances for joint probability between all basic erosion parameters including; wave height, period and direction, event duration, tidal anomalies and event occurrence intensities. This framework includes event grouping where significantly more erosion can occur. This is handled by simulating the event history and estimating the beach accretion between events.

ADDITIONAL INDEX WORDS: *Beach erosion, Generalised extreme values, Generalised Pareto and Poisson distributions*

INTRODUCTION

Coastal zone managers are increasingly seeking accurate quantitative predictions of the beach erosion hazard within a probabilistic framework. This avoids the ad hoc approach in which a standard event (usually the largest measured historical event) is applied and beach erosion and inundation assessed, with little information provided regarding the return period and confidence limits associated with the predictions. That is, in systems involving two or more random variables the return period of outcomes is not equal to the forcing return period of a particular variate (HAWKES *et al.*, 2002). For example, using a 100-year wave height historical event (with unknown storm duration return period) does not guarantee 100-year beach erosion. One reason for this, among others, is the dependency of erosion volumes on storm duration. KRIEBEL and DEAN (1993) showed that during erosion events, there was a finite time required for the new equilibrium profile to form. That is, short duration events having the same peak wave height result in less beach erosion.

Another problem in applying a standard event is that it excludes the merging of independent forcing events (i.e., waves and water surges) from a meteorological view into one beach erosion event. For example, two events of equal magnitude occurring within a few days generate more erosion than if separated by many months (in which the beach is able to recover from the first erosion event).

This paper presents and applies a new comprehensive method to determine beach erosion hazard using recently developed statistical methods. The method is compared to Narrabeen Beach erosion volume extreme statistics to evaluate the method's accuracy.

Four alternative methods are available to replace the ad hoc application of an event yielding erosion with unknown probability; fitting distributions directly to erosion measurements; the structural variable method (SVM); the joint probability method by integration; and full temporal simulation.

This study reviews these statistical methods for estimating extreme quantities and applies a combination of statistical and physical mechanisms to refine the full temporal simulation approach for estimating extreme beach erosion. This refined method is shown to perform well when compared against field measurements.

REVIEW OF METHODS

This section reviews these methods for determining extreme values for beach erosion. These methods could equally be applied to beach inundation or other hazards.

Fitting distributions directly to measurements

Fitting distributions directly to erosion measurements requires large data sets that are generally not available and excludes building into the extreme predictions future temporal changes (e.g., climate change).

Structural variable method (SVM)

The structural variable method takes the measured wave and water levels and uses the structural function to determine key quantities. These computed estimates are then extrapolated using a fitted extreme value distribution, which ignores information within the structural function above the measurement range. The second

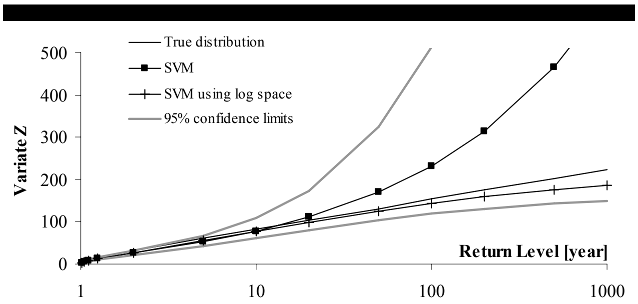

Figure 1. The extreme values for Z where $z = y^2$ and $Y \sim \text{GEV}(5,2,-0.1)$; the true (continuous line) and estimated (continuous line with ■ for SVM and + for SVM applied to the logarithms of Z) distributions. The gray solid lines are the estimated maximum 95% confidence intervals.

weakness of this approach is that temporal changes are limited to the measurement period.

Joint probability method (JPM)

This method applies various statistical modelling techniques to all quantities required by the structural function, including any dependency. These models are formulated into a single joint distribution density function, which is integrated over all combinations of variables ranges that exceed a certain threshold of the structural function to determine the exceedance probability. Hence, information from the structural function above the measurements is included.

This method excludes temporal variations (e.g., sea-level rise or La Niña/El Niña) and the process where independent events are merged into a single event.

Full temporal simulation

This method involves three phases; event simulation using the joint probability and a Poisson process for event occurrence modelling yielding a event history; computing desired quantities using this event history and the structural function; and computing the return levels and confidence intervals of the desired quantities.

With an appropriate Poisson process model, this approach can handle event grouping and has the possibilities of including climate change and non-stationary processes (La Niña/El Niña).

This method does not, however, perform well on non-linear structural functions (e.g., HAWKES, 2000). That is, the final result may not follow the usual extreme value distributions. This is not an issue when return levels are easily determined empirically. However, empirical methods are not able to provide very good extrapolation if future temporal variations are to be included. That is, if future changes are not included long simulation lengths can be used to improve empirical estimates for large return periods. However, if climate changes are included they set the simulation length and consequently limit the return period that can be confidently estimated.

The structural function non-linearity limitation is demonstrated in the following numerical experiment. The random variable Z is estimate from the structure function $z = y^a$, $a = 2$ where Y follows the generalised extreme value (GEV) distribution

$$\Pr\{Y \le y\} = e^{-\left[1+\xi(y-\mu)/\sigma\right]^{-1/\xi}} \quad (1)$$

with μ, σ and ξ being the GEV location, scale and shape parameters (COLES, 2001; JENKINSON, 1955). The numerical experiment simulated Y for 100 years, then extreme estimates for the random variate Z was determined analytically and using the SVM described above. Figure 1 shows SVM compares well up to *ca* 15 year return level. Further testing shows this limitation reduces for $a \in [0.7; 1.2]$, which is weakly non-linear structure function (not shown). One method to overcome this prediction limitation involves analysing the logarithms of Z, which minimises the non-linearities. Figure 1 shows that this transformation extends the accuracy to *ca* 60 year return level, with underestimates produced at higher return levels.

Given that this method, the full temporal simulation, is able to include event grouping and that we can overcome this non-linearity issue, this approach has been adopted for the coastal hazard estimation and is implemented using the following steps;

1. Identify meteorologically independent wave storm events.
2. Fit distributions to wave height and storm duration (marginal distributions).
3. Fit the dependency between wave height and storm duration.
4. Fit the conditional (on wave height) distributions to wave period and peak tidal anomaly.
5. Determine the empirical distribution for wave direction.
6. Fit non-homogenous Poisson distribution to storm occurrences.
7. Simulate the wave climate using the fitted distributions including storm spacing.
8. Determine extreme values of beach erosion from the simulated wave climate.

Steps 1—5 were proposed by HAWKES, *et al.* (2002) and tasks 6—8 are for including the storm clustering. This approach is different from others in that steps 4 and 7 are new.

HAZARD ESTIMATION

In this section, the adopted statistical framework is applied to estimate wave related erosion hazard at Narrabeen Beach, Sydney with the following section comparing result to field measurements.

The offshore wave measurements were obtained from Botany Bay and between North Heads and Long Reef Point, both located near Sydney. The first location has non-directional wave measurements for 35 years while the second site provides wave directions covering 14 years. The tidal anomalies were determined from measurements covering 92 years at Fort Denison which excludes wave set-up and run-up.

Step 1—data preparation

To aid in the identification of possible independent events, periods were identified where significant wave height (H_s) exceeded 3 m (KULMAR *et al.*, 2005; LORD and KULMAR, 2000). These periods were then manually assessed (accepted with or without adjustment or rejected) to ensure the events are meteorologically independent.

Step 2—fitting marginal distribution

Following the method of HAWKES, *et al.* (2002), Generalised Pareto (GP) distributions where fitted to the peak H_s and storm duration (D) using the maximum likelihood method. The GP distribution is

$$\Pr\{X > x | X > u\} = \{1 + \xi(x-u)/\sigma\}^{-1/\xi} \quad (2)$$

with σ and ξ being the GP scale and shape parameters (COLES, 2001) and u being the threshold that ensures parameter convergence and for $H_{s,max}$ (maximum H_s during the event) at Botany Bay, this yields $u = 4.5$ m, and σ and ξ are 0.86 m [0.68; 1.08] and 0.03 [-0.11; 0.24] respectively with 95% confidence ranges provided in the square brackets. Similarly, storm duration

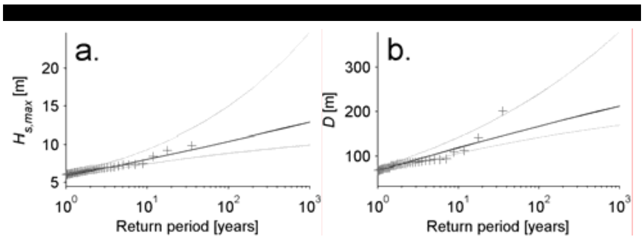

Figure 2. The return level plot for a. maximum H_s and b. D at the Botany Bay wave buoy using GP distribution. Black line, fitted model, gray lines, 95% confidence limits and + are the empirical estimations.

yields $u = 40$ h, $\sigma = 26$ h [20; 32] and $\xi = -0.18$ [-0.29; -0.01]. Figure 2a&b provides the fitted distribution return level plots with 95% confidence limits and the Botany Bay empirical estimates using $\Pr(x_i) = i/(N+1)$, where $x_1 \le x_2 \le x_3 ... \le x_N$.

Step 3—fitting dependency model

The logistics model (TAWN, 1988),

$$\Pr\left\{X \le x, Y \le y\right\} = e^{-\left[x^{-\frac{1}{\alpha}}+y^{-\frac{1}{\alpha}}\right]^{\alpha}} \qquad (3)$$

where x and y are the rescaled Fréchet variates and α is the dependent parameter, was used for dependency modelling between $H_{s,max}$ and D. When $\alpha = 1$, (3) reduces to x and y being independent variables and when $\alpha = 0$, (3) reduces to perfectly dependent variables. This parameter was estimated by maximising the model likelihood, which yielded $\alpha = 0.64$ [0.58; 0.69].

Step 4—fitting conditional models

Both tidal anomaly (R) and significant wave period (T_s) are conditionally related to $H_{s,max}$ for the following physical reasons; tidal anomalies are often produced by the same meteorological feature; wave period is governed by physical mechanisms that limits its range (e.g., for a stable H_s, wave steepness restricts the possible range that T_s can have) and as wave height is typically the dominant parameter for beach erosion, it was chosen as the conditional parameter.

The east coast low meteorological feature accounts for *ca* 40% of the wave climate at Sydney (SHORT and TRENAMAN, 1992). This meteorological event will locally generate waves and provide onshore winds and low atmospheric pressures near the coast with the last two characteristics able to generate positive tidal anomalies. The maximum tidal anomaly measured at Fort Denison during wave storm events does weakly correlate to $H_{s,max}$ with 30% of the variance explained by a linear relationship. The marginal GP distribution for R was convergent using $u \ge 0.175$ m with $\sigma = 0.06$ m [0.04; 0.07] and $\xi = 0.11$ [-0.04; 0.32]. The logistics model was again used for dependency of R with $H_{s,max}$ with $\alpha = 0.74$ [0.69; 0.8], as expected given the underlying physical mechanisms relating these variates.

The proposed conditional modelling for wave period uses a log-normal distribution

$$\Pr\{T_s = x\} = \left\{\left(x-\kappa\right)\sigma\sqrt{2\pi}\right\}^{-1} e^{-\frac{1}{2}\left(\frac{\ln\left(x-\kappa\right)-\mu}{\sigma}\right)^2} \qquad (4)$$

where model parameters κ, μ and σ are dependent on $H_{s,max}$. The minimum significant wave period (T_s) measured at Botany Bay is $T_{s,min} \approx 3.9\,H_s^{0.5}$ (H_s in metres and $T_{s,min}$ in seconds), which represents the practical maximum storm wave steepness of

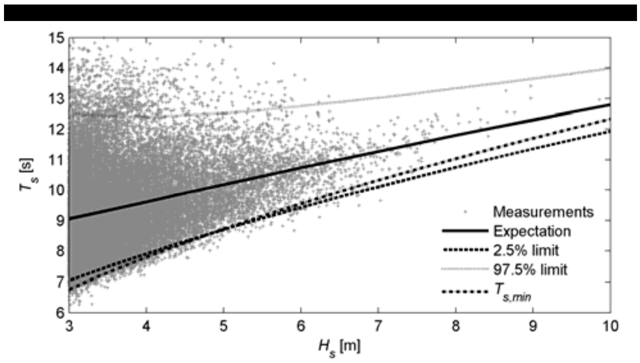

Figure 3. Measured T_s and H_s (+) at Botany Bay where $H_s > 3$ m, the T_s expectation (▬), the 2.5% (┅) and 97.5% (┈) distribution limit for the log-normal model with parameter dependencies according to equation (6) and the estimated lower limit for T_s (┅) using the measurements.

ca 0.04. Consequently, the dependency between model parameters and expectation should ensure most random realisations of T_s exceed this limit. To model this, we situate the expectation as

$$E\left(T_s\right) = \kappa + e^{\mu+\frac{\sigma^2}{2}} \sim k_1 H_s^{k_2} \qquad (5)$$

where k_1 and k_2 are constants. Consequently, the suggested dependency on H_s that is similar to (5) is

$$\left(\kappa,\mu,\sigma\right) = \left(aH_s^b, \ln cH_s^d, \sqrt{2\ln fH_s^g}\right) \qquad (6)$$

yielding an expectation of

$$E\left(T_s\right) = aH_s^b + cfH_s^{d+g} \qquad (7)$$

and applying this to Botany Bay measurements using the maximum likelihood method results in $a = 3.005$ [3.001;3.01], $b = 0.543$ [0.542;0.544], $cf = 5.41$ [5.39;5.44] and $d + g = -0.371$ [-0.375;-0.368] when H_s and T_s are in metres and seconds respectively and the 95% confidence intervals are provided in the square brackets. Figure 3 shows the wave measurements, the conditional probability model and $T_{s,min}$, with the model reasonably reproducing the lower limit on wave period.

Step 5—Empirical models

The wave direction is not measured by the offshore Botany Bay wave buoy. The nearby wave buoy located between North Heads and Long Reef provides wave directions, albeit over a shorter interval. Both buoys are located in similar water depth and at these depths, bathymetry contours are reasonably parallel. Figure 4 shows the $H_{s,max}$ during the overlapping period, indicating similar but not exact offshore conditions are measured with the linear relationship $H_{s,max,Botany\ Bay} = 1.04 H_{s,max,Sydney}$ explaining *ca* 70% of the dependency. Consequently, it is reasonable to use Sydney measurements of H_s and peak spectral wave direction (θ_p) to determine the empirical wave direction distribution.

Figure 5 shows the empirical cumulative probability for θ_p limited to the wave direction range measured. Investigation into extreme H_s and θ_p did not yield a significant dependency between these variables (not shown) and consequently, θ_p was adopted as independent to H_s.

Step 6—storm occurrence model

This step investigates the timing between consecutive wave storms and the possibility for storms being grouped together. This

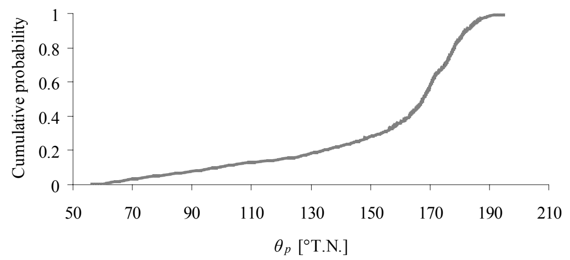

Figure 5. The empirical cumulative probability for θ_p at Sydney measured clockwise from true north (N.T.).

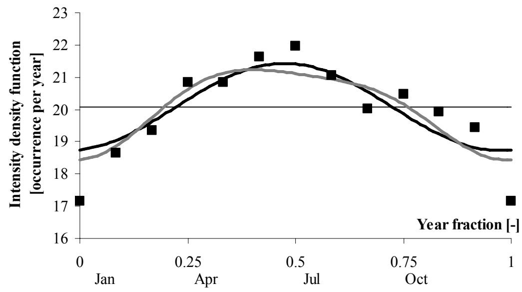

Figure 6. Wave event occurrence at Botany Bay for; measured (■) and modelled (Poisson) with constant (—), annual (▬) and twice-annual (▬) variations in event occurrence intensity.

is of particular interest as there is a potential that two minor storm events occurring within a few days of each other may yield more extensive beach erosion than a single large event. Characterising this event grouping and the seasonal changes in event occurrence intensity from the different meteorological processes generating the waves (SHORT and TRENAMAN, 1992) is essential for a physically plausible Monte Carlo simulation.

Figure 6 shows the measured seasonal trends in occurrences of extreme wave storms with the propensity for more winter events. The seasonal variation in storm occurrence, while ignoring storm grouping, was modelled using a non-homogeneous Poisson process (COLES, 2001; DAVISON and SMITH, 1990; LUCEÑO *et al.*, 2006). Three seasonal occurrence intensity variations were tested; constant, annual and twice-annual. The non-homogeneous Poisson process is defined using the following notation; if the initial event occurs at t_0 and subsequent events at times $t_1, t_2,\ldots,t_n$, with occurrence intensity $\lambda\left(t|\boldsymbol{\theta}\right)$, the log-likelihood function is

$$\ell\left(\mathrm{t}|\boldsymbol{\theta}\right) = -\int_{t_0}^{t_n}\lambda\left(t\right)\mathrm{d}t + \sum\nolimits_{i=1}^{n}\lambda\left(t_i\right) \qquad (8)$$

with $\lambda(t)=\theta_0$ for constant, $\lambda(t)=\theta_0+\theta_1\sin\omega t+\theta_2\cos\omega t$ for annual and $\lambda(t)=\theta_0+\theta_1\sin\omega t+\theta_2\cos\omega t+\theta_3\sin 2\omega t+\theta_4\cos 2\omega t$ for twice-annual occurrence intensity variations and ω is the radian frequency for a one year period ($\omega \approx 1.99\times10^{-7}$ rad/s). Maximising (8) (see figure 6) indicates that the annual model is better at the 54% confidence level over the constant model, with the twice-annual model being better at the 4% confidence level over the annual model. Consequently, the annual model was adopted with ($\hat{\theta}_0,\hat{\theta}_1,\hat{\theta}_2$) = (20.1,0.84,1.05). To test event clustering we follow LUCEÑO *et al.* (2006) approach with constant, annual and twice-annual seasonal event occurrence intensity. Their model introduces two addition parameters, γ_0 and γ_1, and with $\gamma_0 = 0$ being equivalent to the non-clustered model. Maximising (8) with LUCEÑO *et al.* (2006) yields $\gamma_0 = 0$ for the three seasonal models.

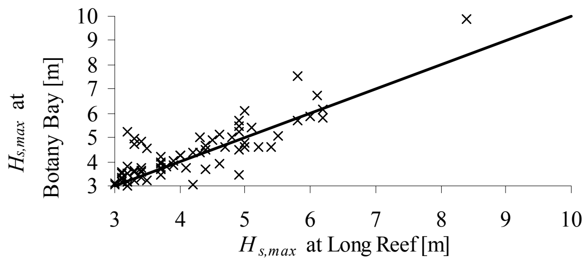

Figure 4. Measured peak storm $H_{s,max}$ (×) for Botany Bay and Long Reef wave buoys for all simultaneously events. The solid continuous line corresponds to identical $H_{s,max}$ at the two stations.

Hence, the measurements do not indicate event clustering. This does not preclude the scenario of two storms occurring close together and consequently producing severe erosion. However, it does imply that there is no evidence within the measurements of temporal dependency other than that already included in the Poisson process adopted.

Step 7—simulation

The extreme wave climate is simulated with the first event at time *t*, using Monte Carlo techniques and the fitted distributions by repeating the following; A)at *t*, calculate tide; B)generate random realisations of $H_{s,max}$, D, T, θ_p and R; C)estimate beach erosion; D)generate time to next storm; and E)determine profile recovery. The KRIEBEL and DEAN (1993) simple time-dependent equilibrium profile model, used in steps C and E, was modified to include non-equilibrium initial conditions, i.e.,

$$V\left(t\right)=\begin{cases} V_i e^{-\frac{t}{T_{s,a}}} & 0\le t\le t_s \\ \dfrac{V_\infty}{T_{s,e}}\displaystyle\int_0^t f\left(\tau\right)e^{-\frac{t-\tau}{T_{s,e}}}\mathrm{d}\tau & t_s < t\le t_m \\ V\left(t=t_m\right)e^{-\frac{t-t_m}{T_{s,a}}} & t>t_m \end{cases} \qquad (9)$$

with $f(t) = \sin^2 \pi t/D$ where t is time measured from the start of a particular storm; V_i and V_∞ are the initial and maximum eroded sand volumes with V_∞ depending on the equilibrium constant A_{eq} (DEAN and DALRYMPLE, 2002); $T_{s,e}$ and $T_{s,a}$ are the characteristic time scale of the exponential response under erosive (storm) and accretive (non-storm) conditions; t_s is the time when storm erosion is greater than the initial erosion; t_m is the time of maximum erosion during the storm and $f(t)$ is the storm shape function. Figure 7 shows an application of (9) in which two independent meteorological events (waves) are merged into one erosional event.

RESULTS

The Narrabeen Beach (see HENNECKE *et al.*, 2004 for a recent site description) profile measurements conducted from 27th April, 1976 until 2nd March, 2006 were used to assess this approach. The profile surveys were conducted on average every 1.5 months and allowed for the estimation of yearly maximum change in beach volume above mean sea level (MSL). Figure 8 shows that the proposed method was able to reproduce the measured extreme behaviour at Narrabeen within a factor 2 for up to 10-year return period. This is remarkable in that the structural function used in

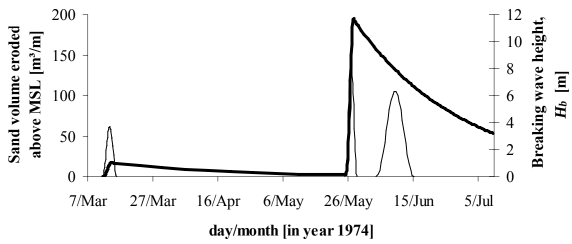

Figure 7. Example application of (9) showing the breaking wave height (—) and exponential beach erosion response using $A = 0.2$ m^{⅓} and $T_s \in$ [8; 12] hours (▬) during the May/June 1974 storms (FOSTER *et al.*, 1975).

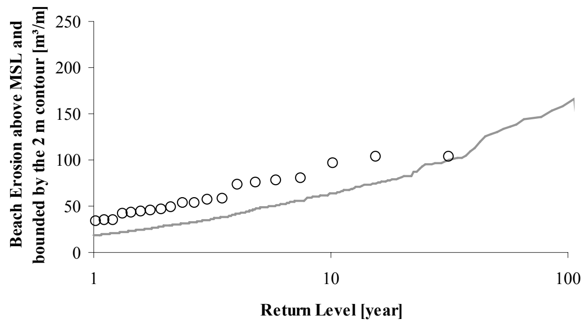

Figure 8. The eroded sand volume above MSL at Narrabeen Beach from; profile measurements (○); and using steps 1-8 with 850 years being simulated (▬).

this article is an exceptionally simple erosion/accretion model. It should be noted that the Narrabeen Beach measurements cover approximately 30 years. Hence, the estimation of extreme quantities (○ in figure 8) are well predicted up to *ca* 7.5 years with subsequent predictions heavily effected by sampling error (i.e., the population is not well represented for return periods exceeding 7.5 years).

DISCUSSION

Sandy coastline management often requires erosion and inundation hazard quantification. The ad hoc approach in which a standard event was used has been replaced by a method that includes event merging and can incorporate climate change. This method was demonstrated using climate information (waves and tides) obtained at Sydney and the simple erosion/accretion model after KRIEBEL and DEAN (1993). Comparison with the extreme beach erosion volumes at Narrabeen Beach confirmed that the statistical approach is able to reproduce the measurements. The comparisons were particularly robust for return periods less than 10 years.

The method presented here provides a solid and defendable framework for determining coastal erosion hazard due to ephemeral wave storms on the NSW coastline. Note, this method does not include chronic and ongoing erosion sources. Extrapolation to extreme return levels using this new framework, while more robust than the ac hoc method involving historical events, should be done with caution.

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